

EFFICIENT APPROACHES TO SOURCE LOCALIZATION
AND PARAMETER ESTIMATION

By

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*To the Memory of My Mother,
Yu-Xian Liu*

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KEY TO ACRONYMS

AOA: Angle-Of-Arrival

AR: Autoregressive

ARMA: Autoregressive Moving Average

AP: Alternating Projection

ANPA: Alternating Notch-Periodogram Algorithm

CCD: Co-centered Crossed Dipoles

COLD: Co-centered Orthogonal Loop and Dipole

CRB: Cramer-Rao Bound

CSM: Coherent Signal-subspace Method

DFT: Discrete Fourier Transform

DS-CDMA: Direct-Sequence Code-Division Multiple Access

EM: ElectroMagnetic

ESPRIT: Estimation of Signal Parameters via Rotational Invariance Techniques

FFT: Fast Fourier Transform

IQML: Iterative Quadratic Maximum Likelihood

LS-ESPRIT: Least-Squares based ESPRIT

LSML: Large Sample Maximum Likelihood

ME: Maximum Entropy

ML: Maximum Likelihood

MODE: Method Of Direction Estimation

MUSIC: Multiple Signal Classification

NLS: Nonlinear Least-Squares

NSF: Noise Subspace Fitting

RELAX: RELAXation algorithm for the minimization of the NLS criterion

RMSE: Root-Mean-Squared Error

SNR: Signal-to-Noise Ratio

SSF: Signal Subspace Fitting

TLS-ESPRIT: Total Least-Squares based ESPRIT

ULA: Uniform Linear Array

WSF: Weighted Subspace Fitting

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This dissertation considers the problem of source localization and parameter estimation with antenna arrays. The problem is to estimate the parameters of the incident electromagnetic plane waves with antenna arrays. Our focus is on array geometry, sensor characteristics, narrowband and wideband signals, and estimation algorithms for the estimation problem.

In particular, the RELAX algorithm, recently proposed for temporal spectral analysis, is extended to solve the spatial problem of angle and waveform estimation for both narrowband and wideband plane waves arriving at a uniform linear array. Unlike

most existing high resolution algorithms, the narrowband and wideband RELAX algorithms are robust against the presence of unknown spatially colored noise. Further, the wideband RELAX algorithm does not need the initial angle estimates that existing wideband algorithms require to construct focusing matrices, which causes biases in angle estimates. The wideband RELAX algorithm naturally focuses the narrowband components in the spatial frequency domain. Both numerical and experimental examples are used to demonstrate the performance of the RELAX algorithm and compare the performance of RELAX with that of other well-known algorithms including ESPRIT with forward/backward spatial smoothing, MODE/WSF, and AP/ANPA for narrowband signals; and CSM-ESPRIT for wideband signals. We also show that better parameter estimates can be obtained by using RELAX as compared to using those other algorithms by means of both numerical and experimental results.

To exploit the advantages of array geometry and antenna sensors, a polarization sensitive linear array that consists of Co-centered Orthogonal Loop and Dipole (COLD) pairs are proposed for the estimation of the parameters of completely polarized narrowband electromagnetic plane waves. The performance of both angle and polarization estimation using the COLD array are shown to be greatly improved as compared to using a crossed dipole array. A MODE algorithm is presented for both angle and polarization estimation of correlated (including coherent) or uncorrelated incident signals with a COLD array. Numerical examples are given to show the better estimation performance of the MODE algorithm as compared to those of the MUSIC and NSF algorithms. Finally, we devise a RELAX approach for angle, polarization

and waveform estimation of narrowband signals with a COLD array. We also use numerical examples to demonstrate the superior performance of RELAX as compared to MODE when the additive noise is spatially colored and unknown.

CHAPTER 1 BACKGROUND AND MOTIVATION

1.1 Background

Many theoretical studies on parameter estimation with an array of sensors have been carried out and deep insight has been achieved in the past two decades. The research in sensor array processing was originally motivated by its applications in source localization and interference suppression in radar and sonar. Many algorithms have appeared in the literature for estimating signal parameters from the measurement output of a sensor array.

The methods in sensor array processing can be classified into two categories: nonparametric methods and parametric methods. The nonparametric methods do not make any assumption on the statistical properties of the data. Spatial filtering techniques [1, 2] are the early approaches to perform a space-time processing of data sampled at an array of sensors. Beamforming and Capon's methods [3, 4, 5, 6, 7] are the typical nonparametric methods. Their idea is to form some spectrum-like function of the parameter(s) of interest, and then take the locations of the highest peaks of the function as the estimates. All these approaches have an inherent limitation of poor resolution. They are usually used in situations where the information about the statistical properties are not available. However, spatial filtering methods with an increasing number of novel applications inspired much of the subsequent

efforts in statistical signal processing. The well-known Maximum Entropy (ME) spectral estimation method in geophysics by Burg [8] and the Yule-Walker autoregressive estimation method are the early parametric approaches. The introduction of subspace-based techniques [9, 10, 11] provided a new geometric interpretation for the sensor array processing problem. The vector space formulation of the sensor array problem resulted in a large number of algorithms [12, 13, 14, 15]. The Maximum Likelihood (ML) parameter estimates can also be derived for the sensor array problem in an appropriate statistical framework. ML estimation is a systematic approach to many parameter estimation problems, and has been studied by many researchers [16, 17, 18, 19, 20, 21, 22]. Unfortunately, the ML method usually requires a multidimensional non-linear optimization search at a considerable computational complexity. Reduced computational cost is generally achieved by the use of a suboptimal estimator. Much work on devising a family of suboptimal estimators and the analysis of the performance of the estimators has been done [23, 24, 25, 26, 27, 28]. Most of these algorithms were motivated by the subspace based technique introduced in the MUSIC algorithm. The subspace-based approach results in a resolution that is not limited by the array aperture, provided that the number of data samples or the signal-to-noise ratio (SNR) is sufficient large. A review of the development of array signal processing algorithms will given in the next chapter.

1.2 Motivation and Contributions

Array signal processing was centered on the ability to fuse data from data acquisition systems, such as an array of sensors, to carry out a given estimation task. Many

sophisticated estimation algorithms were the results of an attempt of researchers to go beyond the classical Fourier-limit. However, many high resolution array processing algorithms are usually very sensitive to the presence of unknown spatially colored noise. Thus the performance of those algorithm is frequently very poor when used in practical applications. To derive more robust algorithms, we need to relax the additive white noise assumption made by most existing high resolution algorithms.

We propose a RELAX (RELAXation algorithm for the minimization of the NLS criterion) algorithm for angle and waveform estimation of narrowband plane waves arriving at a uniform linear array. The RELAX algorithm is robust against the presence of unknown spatially colored noise. Since most of the algorithms in array processing are devised only for narrowband signals, the algorithm development for wideband signals has only received some limited attention. Although some subspace-based methods [29, 30] were introduced for the wideband case, most of them use focusing matrices, which may result in biased estimates. We propose a wideband RELAX algorithm for angle estimation of wideband plane waves arriving at a uniform linear array. Unlike other wideband algorithms, the wideband RELAX does not need initial angle estimates and naturally focuses the narrowband components in the spatial frequency domain.

It is also important to take advantage of array geometries and receiving properties of antenna elements. Although many algorithms have been developed for array signal processing recently, the characteristics of specific antenna sensors are only

beginning to attract more attention. Previous work on angle and polarization estimation using crossed dipoles [31, 32] and orthogonal dipoles and loops [33] are examples of using specific antenna sensors to estimate the angles and polarizations of incident narrowband electromagnetic plane waves. To exploit the advantages of array geometry and antenna sensors, a polarization sensitive linear array that consists of Co-centered Orthogonal Loop and Dipole (COLD) pairs are investigated for the problem of parameter estimation of completely polarized electromagnetic (EM) plane waves. The performance of both angle and polarization estimation using the COLD array are shown to be greatly improved as compared to using a crossed dipole array. We propose an efficient MODE (Method Of Direction Estimation) algorithm for both angle and polarization estimation of correlated (including coherent) or uncorrelated completely polarized narrowband signals with a COLD array. We also devise a RELAX algorithm that can be used with a uniform linear COLD array for angle, polarization and waveform estimation of completely polarized narrowband plane waves.

The high resolution parameter estimation approaches we develop in this dissertation can be used for source localization in radar and sonar as long as incident waves can be modeled as having discrete angles of arrival. For example, in radar applications, the radar returns of targets impinging on the antenna array of a radar receiver enable estimation of the directions of targets with these high resolution parameter estimation algorithms. The parameters of interest include angles-of-arrival (AOAs), waveforms, and polarization states of the incident signals.

1.3 Outline of the Dissertation

Chapter 2 provides the background material for the remainder of the dissertation. The material includes brief reviews of electromagnetic wave propagation and polarization, and the sensor array processing algorithms. This chapter also presents the fundamental sensor array data models for the dissertation.

Chapter 3 describes how the RELAX algorithm can be used for angle and waveform estimation of narrowband signals with a uniform linear array for the case of multiple snapshots. To evaluate the performance of the RELAX algorithm, we apply it to both simulated and experimental data, and compare its performance with that of other well-known algorithms including ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques) with forward/backward spatial smoothing, MODE/WSF (Weighted Subspace Fitting), and AP (Alternating Projection)/ANPA (Alternating Notch-Periodogram Algorithm) for narrowband signals.

In Chapter 4 we extend the RELAX algorithm to the case of wideband sources and also present numerical and experimental examples to illustrate the performance of the wideband RELAX algorithm and compare its performance with that of CSM (Coherent Signal-subspace Method) based ESPRIT.

In Chapter 5 an arbitrary linear array that consists of Co-centered Orthogonal Loop and Dipole (COLD) pairs is proposed for the problem of statistically efficient estimation of the parameters of completely polarized electromagnetic waves. A MODE

algorithm is presented for both angle and polarization estimation of correlated (including coherent) or uncorrelated incident signals with a COLD array. The performance of both angle and polarization estimation using the COLD array are shown to be greatly improved as compared to using a co-centered crossed dipole (CCD) array. Several numerical examples are presented to compare the MODE algorithm with the MUSIC algorithm and the NSF algorithm for both angle and polarization estimation. The Cramer-Rao bound (CRB) for the COLD array is also compared with that for the CCD array.

Chapter 6 presents a RELAX approach for parameter estimation of narrowband signals arriving at a uniform linear COLD array. The statistical performance of this RELAX estimator is compared with that of the MODE estimator described in Chapter 5 via numerical examples.

Finally, Chapter 7 gives the conclusions and future work.

Parts of the original work presented in this dissertation have already been documented in the publications [34, 35, 36, 37].

CHAPTER 2 WAVE PROPERTIES AND SENSOR ARRAY PROCESSING

2.1 Introduction

In this chapter we provide the groundwork necessary for the material to be developed in the subsequent chapters. After briefly reviewing the important properties of electromagnetic (EM) waves including wave propagation and wave polarization, we introduce the central problem of the dissertation: the problem of locating radiating sources by using an array of passive sensors. Once the data model for the output signal of the receiving sensor array is formed, the source location problem is turned into a parameter estimation problem. The estimation problem has been investigated under both nonparametric and parametric approaches.

2.2 Propagation of EM Waves

Electromagnetic wave propagation is referred to the phenomena that a time-changing electric field produces a time-varying magnetic field, which in turn generates an electric field, and so on with a resulting propagation of energy. The direction of the electric field $\mathbf{E}$ and magnetic field $\mathbf{H}$ are everywhere perpendicular. The most important and most fundamental electromagnetic waves are the transverse plane waves. In a plane wave $\mathbf{E}$ and $\mathbf{H}$ lie in a plane. A wave of this type with both $\mathbf{E}$ and $\mathbf{H}$ transverse to the direction of propagation is called Transverse ElectroMagnetic (TEM) wave. In a medium with spatially constant permeability μ and permittivity

ϵ and with no free charges and currents, the Maxwell's equations are

$$\nabla \mathbf{B} = 0, \quad (2.1)$$

$$\nabla \mathbf{E} = 0, \quad (2.2)$$

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t}, \quad (2.3)$$

$$\nabla \times \mathbf{H} = \epsilon \frac{\partial \mathbf{E}}{\partial t}. \quad (2.4)$$

By combining the two curl equations and making use of vanishing divergences, we can find easily that each cartesian component of $\mathbf{E}$ and $\mathbf{H}$ satisfies the wave equation:

$$\nabla^2 u - \mu \epsilon \frac{\partial^2 u}{\partial t^2} = 0. \quad (2.5)$$

The wave equation has the well-known plane wave solutions

$$u(t, \mathbf{x}) = e^{j(\mathbf{k}^T \mathbf{x} - \omega t)}, \quad (2.6)$$

where the temporal frequency ω and the magnitude of the wave vector $\mathbf{k}$ are related by

$$k = \omega \sqrt{\mu \epsilon}, \quad (2.7)$$

and $\mathbf{x}$ is the position vector.

Clearly, the phase variation of the wave signal $u(t, \mathbf{x})$ includes both temporal and spatial variations. Spatial far-field receiving conditions, which implies plane waves, are assumed throughout the dissertation. The superposition principle is valid if more

than one wave travel through a linear medium. Usually these propagating waves carry information from their sources. The information may include source related signal parameters, such as angle-of-arrival (AOA), signal waveform, signal polarization state, propagation delay, etc. These signal parameters are very important in many applications. For source localization, AOA, signal polarization state and waveform are of special interest. The efficient estimation of these parameters, which is the essence of sensor array signal processing, will be the topic of our main interest in this dissertation.

2.3 Polarization of EM Waves

Polarization describes the orientation of the electric field of a wave. It is advantageous to employ an array of diversely polarized antennas since multiple signals can be resolved on the basis of polarization as well as AOA. To specify the polarization of the waves, we consider sinusoidal waves of the same frequency. In a plane wave traveling along the positive z direction, the electric field generally has both x and y components. The general expression for the electric field of such a wave is then given by

$$\mathbf{E} = (E_1\hat{\mathbf{x}} + E_2e^{j\eta}\hat{\mathbf{y}})e^{-jkz}, \quad (2.8)$$

where E_1 and E_2 are real and η ($-180^\circ \leq \eta \leq +180^\circ$) is the phase angle between x and y components. The corresponding magnetic field is

$$\mathbf{H} = \sqrt{\frac{\epsilon}{\mu}}(-E_2e^{j\eta}\hat{\mathbf{x}} + E_1\hat{\mathbf{y}})e^{-jkz}. \quad (2.9)$$

The phase and relative amplitudes E_1 and E_2 determine the state of polarization. In the most general case of elliptical polarization the polarization ellipse described by E_1 and $E_2e^{j\eta}$, as time progresses, may have any orientation as shown in Fig. 2.1. The line segment OA is the semimajor axis, and the line segment OB is the semiminor axis. The axial ratio is

$$AR = \frac{OA}{OB}, \quad (1 \leq AR \leq \infty). \quad (2.10)$$

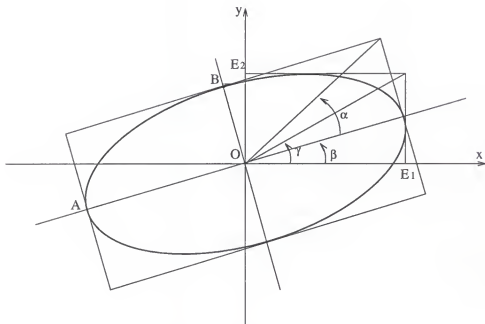


Figure 2.1. Polarization ellipse.

We define β to be the tilt angle of the ellipse ($0 \leq \beta \leq 180^\circ$), and α to be the ellipticity angle, which is given by

$$\alpha = \tan^{-1}(\mp AR), \quad (-45^\circ \leq \alpha \leq +45^\circ). \quad (2.11)$$

The ellipticity angle α is negative for right-handed and positive for left-handed polarization. For the case shown in Fig. 2.1 α is positive. The parameter γ is defined as

$$\gamma = \tan^{-1}\left(\frac{E_2}{E_1}\right), \quad (0^\circ \leq \gamma \leq 90^\circ). \quad (2.12)$$

The geometric relation of α , β and γ to the polarization ellipse is illustrated in Fig.

2.1. The trigonometric interrelations of α , β , γ and η are given by [38, 39]

$$\cos 2\gamma = \cos 2\alpha \cos 2\beta, \quad (2.13)$$

$$\tan \eta = \frac{\tan 2\alpha}{\sin 2\beta}, \quad (2.14)$$

$$\tan 2\beta = \tan 2\gamma \cos \eta, \quad (2.15)$$

$$\sin 2\alpha = \sin 2\gamma \sin \eta. \quad (2.16)$$

The Poincaré sphere representation of wave polarization [40] in Fig. 2.2 clearly shows the relationship among the four angular variables α , β , γ and η . The polarization state is described by a point on a sphere where the longitude and latitude of the point are related to parameters of the polarization ellipse as:

$$\text{Longitude} = 2\beta, \quad (2.17)$$

$$\text{Latitude} = 2\alpha. \quad (2.18)$$

The polarization state described by a point on a sphere can also be expressed in terms of the angle subtended by the great circle drawn from a reference point on

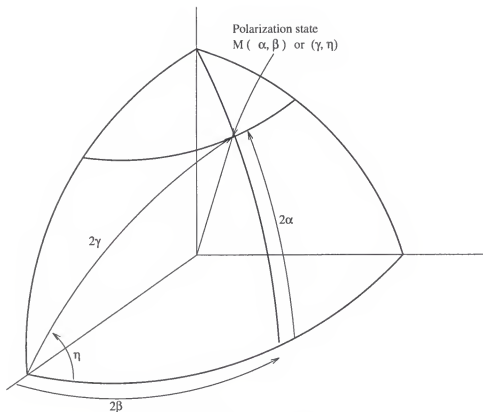


Figure 2.2. Poincaré sphere.

the equator and angle between the great circle and the equator as:

$$\text{Great-circle angle} = 2\gamma, \quad (2.19)$$

$$\text{Equator-to-great-circle angle} = \eta. \quad (2.20)$$

Thus, it is convenient to describe the polarization state by either of the two sets of angles (α, β) or (γ, η) . The case when $\alpha = 0$ corresponds to linear polarization. The case when $\alpha = \pm 45^\circ$ corresponds to circular polarization, with left circular polarization ($\alpha = +45^\circ$) at the upper pole. One octant of the Poincaré sphere and polarization states at specific points are shown in Fig. 2.3. In the general case any point on the upper hemisphere describes a left elliptically polarized wave ranging

from pure left circular at the pole to linear at the equator. Likewise, any point on the lower hemisphere describes a right elliptically polarized wave ranging from pure right circular at the pole to linear at the equator.

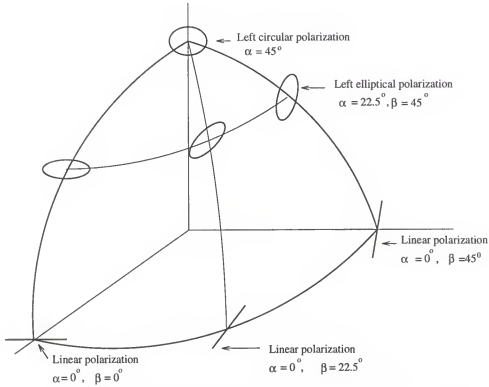


Figure 2.3. One octant of Poincaré sphere with polarization states.

EM waves may be either completely polarized or partially polarized, with the former being the most common. A completely polarized EM wave is a special case of a more general type of EM wave, i.e., a partially polarized EM wave. In other words, the polarization state of a partially polarized EM wave is a function of time while a completely polarized wave has a fixed state of polarization. In practical applications such as radar and ionospheric radio [41, 42], the state of polarization of a returning wave received by a radar with polarization diversity can vary even though the original transmitted wave is completely polarized.

2.4 Narrowband Signals

2.4.1 Array Data Model and Problem Formulation

A general two-dimensional isotropic sensor array system is shown in Figure 2.4. The wave field of the sources travels through space and is sampled, in both space and time, by the sensor array. Assume that the array is planar, each of which has coordinate $\mathbf{r}_l = (x_l, y_l)$ and an impulse response

$$h_l(t, \mathbf{r}_l) = a_l(\theta) \delta(t) \delta(\mathbf{r}_l), \quad l = 1, \dots, L. \quad (2.21)$$

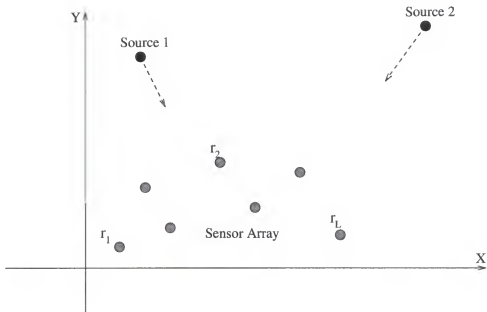


Figure 2.4. An arbitrary two-dimensional array.

For K signals impinging on an array of L sensors, we can define an $L \times K$ impulse matrix $\mathbf{H}(t, \boldsymbol{\theta})$ from the impinging emitter signals with parameter $\boldsymbol{\theta} = [\theta_1, \theta_2, \dots, \theta_K]$, to the sensor outputs. The lk th element of $\mathbf{H}(t, \boldsymbol{\theta})$ takes the

form

$$\mathbf{H}_{lk}(t, \theta) = a_l(\theta_k) \delta(t) \delta(\mathbf{r}_l), \quad l = 1, \dots, L, k = 1, 2, \dots, K. \quad (2.22)$$

From a convolution operation, the sensor outputs can be written as

$$\mathbf{y}_l(t) = \sum_{k=1}^K a_l(\theta_k) e^{j(\omega t - \mathbf{k}_k^T \mathbf{r}_l)} \quad (2.23)$$

for purely exponential signals and

$$\mathbf{y}_l(t) = \sum_{k=1}^K a_l(\theta_k) e^{j(\omega t - \mathbf{k}_k^T \mathbf{r}_l)} s_k(t) \quad (2.24)$$

for narrowband signals, where ω is the common center frequency of the signals and $s_k(t)$ represents the complex amplitude (waveform) of the k th signal. The carrier $e^{j\omega t}$ is usually removed from the sensor output before sampling.

It is clear that the geometry of a given array determines the relative delays of the various angles-of-arrival (AOA). The formulation can be straightforwardly extended to arrays where additional dimensions provide the flexibility for more signal parameters per source, such as a polarization sensitive sensor array for both AOA and polarization estimation. We will introduce a polarization sensitive sensor array later in this dissertation.

The received signal plus noise gives the outputs of the sensor array in the form of

$$\mathbf{y}(t) = \begin{bmatrix} \mathbf{a}(\theta_1), & \mathbf{a}(\theta_2), & \dots, & \mathbf{a}(\theta_K) \end{bmatrix} \mathbf{s}(t) + \mathbf{n}(t) = \mathbf{A} \mathbf{s}(t) + \mathbf{n}(t), \quad (2.25)$$

where

$$\mathbf{a}(\theta_k) = \begin{bmatrix} a_1(\theta_k)e^{j\mathbf{k}_k^T \mathbf{r}_1}, & a_2(\theta_k)e^{j\mathbf{k}_k^T \mathbf{r}_2}, & \dots, & a_L(\theta_k)e^{j\mathbf{k}_k^T \mathbf{r}_L} \end{bmatrix}^T, \quad (2.26)$$

$$\mathbf{s}(t) = \begin{bmatrix} s_1(t), & s_2(t), \dots, & s_K(t) \end{bmatrix}^T. \quad (2.27)$$

and $\mathbf{n}(t)$ is the noise vector. The vector $\mathbf{a}(\theta_k)$ is referred to as the k th array propagation steering vector. It describes via $\mathbf{k}_k^T \mathbf{r}_l$ how a plane wave impinges at the array (i.e. AOA) and via $a_l(\theta_k)$ how the sensors affect the signal amplitude and phase. For polarization sensitive sensors, the $a_l(\theta_k)$ is related to the polarization state of the incident electromagnetic plane wave. For omnidirectional isotropic sensors, the $a_l(\theta_k)$ is a constant. We consider both isotropic sensor arrays (Chapters 3 and 4) and polarization sensitive sensor arrays (Chapters 5 and 6) in this dissertation. The reviews of previous work presented below are for an isotropic sensor array unless we point out a polarization sensitive array.

For a uniform linear array (ULA) shown in Figure 2.5, the steering vector $\mathbf{a}(\theta_k)$ has the form

$$\mathbf{a}(\theta_k) = \begin{bmatrix} 1, & e^{j\Omega_k}, & \dots, & e^{j(L-1)\Omega_k} \end{bmatrix}^T \quad (2.28)$$

where

$$\Omega_k = \frac{\omega}{c} \delta \sin(\theta_k) \quad (2.29)$$

is called the spatial frequency.

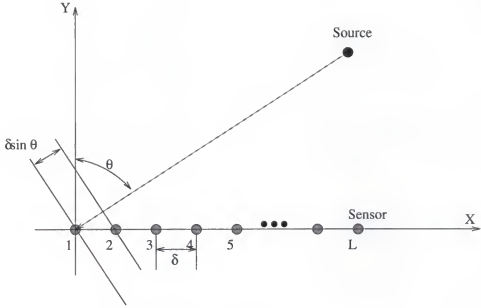


Figure 2.5. A uniform linear array.

We can see from (2.29) that the vector $\mathbf{a}(\theta_k)$ is uniquely defined if and only if Ω_k is constrained as $|\Omega_k| \leq \pi$. The condition is satisfied if

$$\delta \leq \frac{\lambda}{2}. \quad (2.30)$$

The collection of these steering vector over the parameter space of interest

$$\mathcal{A} = \{\mathbf{a}(\theta_k) | \theta_k \in \Theta\} \quad (2.31)$$

is often called the array manifold. The parameterization of $\mathcal{A}$ is assumed known. Let us define the set $\mathcal{A}^K$ as the collection of all distinct array manifold vectors

$$\mathcal{A}^K = \{\mathbf{A} | \mathbf{A} = [\mathbf{a}(\theta_1) \ \cdots \ \mathbf{a}(\theta_K)] , \theta_1 < \theta_2 < \cdots < \theta_K\}. \quad (2.32)$$

Hence, $\mathcal{A}^K$ is parameterized by the parameter vector $\boldsymbol{\theta} = [\theta_1, \dots, \theta_K]^T$. The array is assumed to be unambiguous. In other words, any $\mathbf{A} \in \mathcal{A}^K$ has full rank.

The sensor outputs are appropriately sampled at $t = 1, 2, \dots, N$ time instances and these snapshots $\mathbf{y}(1), \mathbf{y}(2), \dots, \mathbf{y}(N)$ can be viewed as a multichannel random process, which is assumed Gaussian in this dissertation. The characteristics of Gaussian processes can be well understood from its first and second order statistics determined by the underlying signals as well as noise. The problem of central interest for source localization is to estimate the AOAs, waveforms (and polarization states if a polarization sensitive array is used) of emitter signals impinging on a receiving array when a set of sample data $\{\mathbf{y}(1), \mathbf{y}(2), \dots, \mathbf{y}(N)\}$ is given.

We first make some assumptions for the additive noise $\mathbf{n}(t)$ and signal waveforms $\mathbf{s}(t)$. The noise vector $\mathbf{n}(t)$ is assumed to be a stationary, temporally white, zero-mean and circularly symmetric with unknown covariance matrix $\mathbf{Q}$:

$$\mathbb{E}\{\mathbf{n}(t_1)\mathbf{n}^H(t_2)\} = \mathbf{Q}\delta_{t_1, t_2}, \quad (2.33)$$

$$\mathbb{E}\{\mathbf{n}(t_1)\mathbf{n}^T(t_2)\} = 0, \quad (2.34)$$

where δ_{t_1, t_2} is the Kronecker delta, $(\cdot)^H$ represents the complex conjugate transpose, and $(\cdot)^T$ denotes the transpose. Note that the problem of angle estimation is ill-defined for an arbitrary noise field without knowledge of the signal waveform. The eigenstructure-based estimation methods assume the case of spatially white noise, i.e., $\mathbf{Q} = \sigma^2 \mathbf{I}$.

The signal waveform $\mathbf{s}(t)$ is assumed to be deterministic unknown or random.

For the latter case, the source covariance matrix is defined as

$$\mathbf{R}_s = \mathbb{E}\{\mathbf{s}(t)\mathbf{s}^H(t)\}. \quad (2.35)$$

For the former case,

$$\mathbf{R}_s = \frac{1}{N} \sum_{t=1}^N \mathbf{s}(t)\mathbf{s}^H(t). \quad (2.36)$$

For the case of spatially white noise, the array covariance matrix has the form

$$\mathbf{R} = \mathbb{E}\{\mathbf{y}(t)\mathbf{y}^H(t)\} = \mathbf{A}\mathbf{R}_s\mathbf{A}^H + \sigma^2\mathbf{I}. \quad (2.37)$$

The eigendecomposition of $\mathbf{R}$ results in the representation

$$\mathbf{R} = \sum_{i=1}^L \lambda_i \mathbf{e}_i \mathbf{e}_i^H = \mathbf{E}_s \mathbf{\Lambda}_s \mathbf{E}_s^H + \mathbf{E}_n \mathbf{\Lambda}_n \mathbf{E}_n^H, \quad (2.38)$$

where $\lambda_1 > \dots > \lambda_K > \lambda_{K+1} = \dots = \lambda_L = \sigma^2$. The matrix $\mathbf{E}_s = [\mathbf{e}_1, \dots, \mathbf{e}_K]$ contains the K eigenvectors corresponding to the largest eigenvalues. The range space of $\mathbf{E}_s$ is called the *signal subspace*. Its orthogonal complement is the *noise subspace* and is spanned by the columns of $\mathbf{E}_n = [\mathbf{e}_{K+1}, \dots, \mathbf{e}_L]$.

The eigendecomposition of the sample covariance matrix $\hat{\mathbf{R}}$ is given by

$$\hat{\mathbf{R}} = \frac{1}{N} \sum_{t=1}^N \mathbf{y}(t)\mathbf{y}^H(t) = \hat{\mathbf{E}}_s \hat{\mathbf{\Lambda}}_s \hat{\mathbf{E}}_s^H + \hat{\mathbf{E}}_n \hat{\mathbf{\Lambda}}_n \hat{\mathbf{E}}_n^H. \quad (2.39)$$

The number of signals K is assumed known throughout the dissertation. Signal enumeration methods can be found in, e.g., [43, 44, 45, 46, 47, 48].

In the next two subsections, we introduce the most well known estimation techniques, classified as nonparametric and parametric methods.

2.4.2 Nonparametric Methods

The nonparametric methods do not make any assumption on the statistical properties of the data. The fundamental idea of the nonparametric methods is to form some spectrum-like function of the parameter(s) of interest, and then take the location of the highest peaks of the function as the estimates.

The beamforming is to “steer” the array in one direction at a time and measure the array output power. The locations of maximum power yield the AOA estimates. The array is steered by forming a linear combination of the sensor outputs

$$y_F(t) = \sum_{i=1}^L w_i^* y_i(t) = \mathbf{w}^H \mathbf{y}(t). \quad (2.40)$$

Given $\{y_F(t)\}_{t=1}^N$, the output power is measured by

$$\frac{1}{N} \sum_{t=1}^N |y_F(t)|^2 = \frac{1}{N} \sum_{t=1}^N \mathbf{w}^H \mathbf{y}(t) \mathbf{y}^H(t) \mathbf{w} = \mathbf{w}^H \hat{\mathbf{R}} \mathbf{w}. \quad (2.41)$$

Different choices of weighting vector $\mathbf{w}$ leads to different beamforming approaches. There are two types of beamformers.

Conventional Beamformer

The weighting vector for conventional beamformer is given by

$$\mathbf{w}_{BF} = \mathbf{a}(\boldsymbol{\theta}), \quad (2.42)$$

which can be interpreted as a spatial filter matched to the impinging signal.

Substituting the weighting vector (2.42) into (2.41), the beamforming AOA estimates are given by the locations of the largest peaks of the spatial spectrum

$$P_{BF}(\theta) = \mathbf{a}^H(\theta) \hat{\mathbf{R}} \mathbf{a}(\theta). \quad (2.43)$$

Clearly, for a ULA, the beamforming is a spatial extension of the classical periodogram in temporal time series analysis [7]. Thus, the spatial spectrum suffers from the same resolution problem as does the periodogram. This sets a limit on the resolution achievable by beamforming, which is $2\pi/L$ for a ULA.

Capon's Beamformer

A well-known method was proposed by Capon [3], whose beamformer attempts to minimize the power received from noise and any signals coming from other directions than θ , while maintaining a fixed gain in the direction θ . The Capon spatial filter design problem is posed as

$$\min_{\mathbf{w}} \mathbf{w}^H \hat{\mathbf{R}} \mathbf{w} \quad \text{subject to} \quad \mathbf{w}^H \mathbf{a}(\theta) = 1. \quad (2.44)$$

The optimal $\mathbf{w}$ is given by

$$\mathbf{w}_{CP} = \frac{\hat{\mathbf{R}}^{-1}\mathbf{a}(\theta)}{\mathbf{a}^H(\theta)\hat{\mathbf{R}}^{-1}\mathbf{a}(\theta)}, \quad (2.45)$$

which leads to the following spatial spectrum upon insertion into (2.41)

$$P_{CP} = \frac{1}{\mathbf{a}^H(\theta)\hat{\mathbf{R}}^{-1}\mathbf{a}(\theta)}. \quad (2.46)$$

Thus, the Capon AOA estimates are obtained by the locations of the largest peaks of the spatial spectrum given by (2.46).

Capon's beamformer outperforms the conventional one because the former uses every single degree of freedom to concentrate the received energy along the direction of interest. However, the resolution capability of the capon beamformer is still dependent on the array aperture. Since nonparametric methods do not assume anything about the statistical properties of the data, they can be used in situations where we lack information about these properties. Several alternative methods for beamforming have been proposed for addressing various issues [4, 5, 6].

2.4.3 Parametric Methods

When some of the statistical information of the data is available, the use of a nonparametric approach is often associated with a degradation in performance as compared to the use of model-based parametric approach. The most important model-based approach is the Maximum Likelihood (ML) method. The ML estimates are those values of the unknown parameters that maximize the likelihood function.

There are two different ML approaches in the sensor array problem, depending on the model assumption about the signal waveform. The unknown, deterministic signal model leads to the deterministic ML method [18, 17, 49, 50, 51], while the Gaussian random signal model results in the stochastic ML method [19, 52, 53]. Although the ML approaches are optimal to the sensor array problem, these techniques are often deemed exceedingly complex. For the case of spatially white noise, the subspace-based techniques provide many high resolution and computationally efficient algorithms. These algorithms include MUSIC [10, 11] and its modified versions [54, 55, 56, 57], and ESPRIT [58, 59] as well as root-MUSIC [60, 61] for a uniform linear array. The subspace-based techniques also lead to the subspace-based ML approximation approaches, which include Signal Subspace Fitting (SSF) [23, 62, 63] and related, yet more efficient Method Of Direction Estimation (MODE) [25, 26], as well as Noise Subspace Fitting (NSF) [25, 64, 65]. Compared to the exact ML approach, the subspace-based ML approximation approaches are computationally more attractive. However, the subspace-based ML approximation approaches are usually sensitive to the violation of white Gaussian noise assumption, and the number of incident signals. In the next three subsections we will briefly review these important parametric methods.

ML Methods

The ML methods include the deterministic ML and stochastic ML.

- Deterministic ML

The deterministic ML method maximizes the conditional likelihood function (given the signal waveforms and the incident angles) of the data. It is also called the nonlinear least-squares (NLS) method. This leads to the minimization of the following function [18, 17]:

$$NLF = \sum_{n=1}^N [\mathbf{y}(n) - \mathbf{A}(\boldsymbol{\theta})\mathbf{s}(n)]^H [\mathbf{y}(n) - \mathbf{A}(\boldsymbol{\theta})\mathbf{s}(n)]. \quad (2.47)$$

Minimizing the cost function with respect to $\mathbf{s}(1), \mathbf{s}(2), \dots, \mathbf{s}(N)$ gives

$$\hat{\mathbf{s}}(t) = (\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H \mathbf{y}(t), \quad t = 1, 2, \dots, N. \quad (2.48)$$

By substituting (2.48) in (2.47), we get

$$\hat{\boldsymbol{\theta}} = \frac{1}{N} \sum_{t=1}^N \|\{\mathbf{I} - \mathbf{A}(\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H\} \mathbf{y}(t)\|^2 \quad (2.49)$$

$$= \arg \max_{\boldsymbol{\theta}} \text{Tr}\{\mathbf{A}(\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H \hat{\mathbf{R}}\}, \quad (2.50)$$

where $\hat{\mathbf{R}}$ is the sample covariance matrix.

Note that a nonlinear K -dimensional optimization problem must be solved for the DML estimator. A good initial estimate is important to guarantee the desired global minimum since the criterion function often possesses a large number of local minima. A spectral-based method is a natural choice for an initial estimator provided all sources can be resolved. The alternating projection (AP) [66] and alternating notch-periodogram algorithm (ANPA) [67] are two similar efficient approaches to

minimize the nonlinear least-squares criterion. The IQML algorithm [49, 50, 51] is an iterative procedure to minimize the deterministic ML criterion in (2.50) for a uniform linear array. The RELAX algorithm presented in the next chapter is a FFT-based method to minimize the NLS criterion in (2.49).

- Stochastic ML

The Gaussian signal model assumption not only is a way to obtain a tractable ML method, but also is often motivated by the Central limit Theorem. Assume the signal waveforms are zero-mean with second-order properties

$$\mathbb{E}\{\mathbf{s}(t_1)\mathbf{s}^H(t_2)\} = \mathbf{R}_s\delta_{t_1,t_2}, \quad \mathbb{E}\{\mathbf{s}(t_1)\mathbf{s}^T(t_2)\} = 0. \quad (2.51)$$

Under the assumption, the observation process $\mathbf{y}(t)$ is a white, stationary, zero-mean and circularly symmetric Gaussian random vector with covariance matrix

$$\mathbf{R} = \mathbf{A}(\theta)\mathbf{R}_s\mathbf{A}^H(\theta) + \sigma^2\mathbf{I}. \quad (2.52)$$

The negative log-likelihood function (ignoring constant terms) of the complete data set $\mathbf{y}(1), \dots, \mathbf{y}(N)$ is given by

$$l(\theta, \mathbf{R}_s, \sigma^2) = \log|\mathbf{R}| + \text{tr}\{\mathbf{R}^{-1}\hat{\mathbf{R}}\}. \quad (2.53)$$

Minimization with respect to σ^2 and $\mathbf{R}_s$ leads to the following expressions:

$$\hat{\mathbf{R}}_s(\theta) = \mathbf{A}^\dagger(\hat{\mathbf{R}} - \hat{\sigma}^2(\theta)\mathbf{I})\mathbf{A}^{\dagger H}, \quad (2.54)$$

$$\hat{\sigma}^2(\boldsymbol{\theta}) = \frac{1}{L-K} \text{tr}\{\mathbf{P}_A^\perp \hat{\mathbf{R}}\}. \quad (2.55)$$

where

$$\mathbf{A}^\dagger = (\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H, \quad (2.56)$$

$$\mathbf{P}_A^\perp = \mathbf{I} - \mathbf{A} (\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H. \quad (2.57)$$

Substituting (2.54) and (2.55) into (2.53) results in the following compact form of the criterion:

$$\hat{\boldsymbol{\theta}} = \arg \min_{\boldsymbol{\theta}} \left\{ \log |\mathbf{A} \hat{\mathbf{R}}_s(\boldsymbol{\theta}) \mathbf{A}^H + \hat{\sigma}^2(\boldsymbol{\theta}) \mathbf{I}| \right\}. \quad (2.58)$$

The dimension of the parameter space is reduced substantially for the SML estimator. However, the criterion function in (2.58) is a highly non-linear function of $\boldsymbol{\theta}$, and the minimization $\boldsymbol{\theta}$ cannot be found analytically in general. A Newton-type implementation of numerical search may result in a very good accuracy when the global minimum is attained. In fact the stochastic ML estimator has been shown to have a better large sample accuracy than the corresponding deterministic ML estimator [20, 62] regardless of the actual distribution of the signal waveforms.

Subspace-Based Methods

The tremendous interest in the subspace approach is mainly the result of the introduction of the Multiple Signal Classification (MUSIC) algorithm [68, 60], although Pisarenkos work (a special case of MUSIC) [69] in time series analysis was published in early 70's. These methods assume that the additive noise is either spatially white

or its covariance matrix is known. Our discussions on the subspace-based methods below assume that the noise is spatially white.

- MUSIC

The MUSIC algorithm is based on the fact that the noise eigenvectors in $\mathbf{E}_n$ are orthogonal to $\mathbf{A}$, i.e.,

$$\mathbf{E}_n^H \mathbf{a}(\theta) = 0, \quad \theta \in \{\theta_1, \dots, \theta_K\}. \quad (2.59)$$

For unique AOA estimates, the array is usually assumed to be unambiguous. In particular, a ULA is unambiguous if $-90^\circ < \theta < 90^\circ$ and $d < \lambda/2$. In practice, the MUSIC “spatial spectrum” is defined as

$$P_M(\theta) = \frac{\mathbf{a}^H(\theta) \mathbf{a}(\theta)}{\mathbf{a}^H(\theta) \hat{\mathbf{E}}_n \hat{\mathbf{E}}_n^H \mathbf{a}(\theta)}. \quad (2.60)$$

The MUSIC AOA estimates are given by the locations of the largest peaks of the “spatial spectrum” in (2.60) provided $\hat{\mathbf{E}}_n \hat{\mathbf{E}}_n^H$ is close to $\mathbf{E}_n \mathbf{E}_n^H$.

In contrast to the beamforming techniques, the MUSIC algorithm provides estimates of an arbitrary accuracy if the data samples are sufficiently large or the SNR (signal-to-noise ratio) is adequately high. A number of modifications of MUSIC have been proposed to improve or overcome some of its shortcoming in various specific scenarios. The Min-Norm algorithm [54, 55, 56, 57], a weighted MUSIC, exhibits a better resolution than the original MUSIC algorithm [70].

- ESPRIT

A uniform linear array steering matrix has the structure of a Vandermonde matrix as follow

$$\mathbf{A} = \begin{bmatrix} 1 & \cdots & 1 \\ e^{j\Omega_1} & \cdots & e^{j\Omega_K} \\ \vdots & \vdots & \vdots \\ e^{j(L-1)\Omega_1} & \cdots & e^{j(L-1)\Omega_K} \end{bmatrix} \quad (2.61)$$

Making use of this special structure results in several computationally and statistically efficient subspace-based algorithms. This type of algorithm may include ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques) [58, 59], and Root-MUSIC [60] that will be introduced later.

The ESPRIT algorithm exploits a so-called shift structure of the Vandermonde matrix $\mathbf{A}$. let $\mathbf{A}_1$ and $\mathbf{A}_2$ be the sub-matrices by deleting the first and last rows from $\mathbf{A}$ respectively. Then, $\mathbf{A}_1$ and $\mathbf{A}_2$ are related by

$$\mathbf{A}_2 = \mathbf{A}_1 \Phi, \quad (2.62)$$

where

$$\Phi = \text{diag}\{e^{j\Omega_1}, e^{j\Omega_2}, \dots, e^{j\Omega_K}\}. \quad (2.63)$$

Consider the structure of the eigendecomposition of the array covariance matrix $\mathbf{R}$ given in (2.38). If the signal covariance matrix $\mathbf{R}_s$ has rank K' , then the matrix $\mathbf{E}_s$ will span a K' -dimensional subspace of $\mathbf{A}$. This observation implies that there exists

a full-rank $K \times K'$ matrix such that

$$\mathbf{E}_s = \mathbf{A}\mathbf{T}. \quad (2.64)$$

Deleting the first and last rows of (2.64), respectively, gives

$$\mathbf{E}_{s1} = \mathbf{A}_1\mathbf{T}, \quad \mathbf{E}_{s2} = \mathbf{A}_2\mathbf{T}. \quad (2.65)$$

Combining (2.62) and (2.65) leads to

$$\mathbf{E}_{s2} = \mathbf{E}_{s1}\mathbf{\Psi}, \quad (2.66)$$

where $\mathbf{\Psi} = \mathbf{T}^{-1}\mathbf{\Phi}\mathbf{T}$. Clearly, $\mathbf{\Psi}$ and $\mathbf{\Phi}$ have the same eigenvalues. The eigenvalues are given by $e^{j\Omega_k}$, $k = 1, 2, \dots, K$, which are related to AOAs. There are two different ESPRIT algorithms, depending on how to approximate the relation

$$\hat{\mathbf{E}}_{s2} = \hat{\mathbf{E}}_{s1}\hat{\mathbf{\Psi}}. \quad (2.67)$$

Solving the approximation relation (2.67) in a least-squares sense results in LS-ESPRIT, while in a Total-Least-Squares sense leads to TLS-ESPRIT.

- Root-MUSIC

The idea of the Root-MUSIC dates back to Pisarenko's method [69]. If the signals covariance matrix has full rank, then the polynomial

$$p_l(z) = \mathbf{e}_l^H \mathbf{a}(z), \quad l = K + 1, \dots, L, \quad (2.68)$$

where $\mathbf{a}(z) = [1, z, \dots, z^{L-1}]^T$, has K zero-roots at $e^{j\Omega_k}$, $k = 1, 2, \dots, K$. Consider the zeros of the MUSIC function

$$\|\hat{\mathbf{E}}_n^H \mathbf{a}(z)\|^2 = \mathbf{a}^H(z) \hat{\mathbf{E}}_n \hat{\mathbf{E}}_n^H \mathbf{a}(z). \quad (2.69)$$

The search for zeros is complicated since the function is not a polynomial in z . In fact, we are only interested in values of z on the unit circle. This suggests that we can use $\mathbf{a}^T(z^{-1})$ to substitute for $\mathbf{a}^H(z)$, which gives the Root-MUSIC polynomial

$$p(z) = z^L \mathbf{a}^T(z^{-1}) \hat{\mathbf{E}}_n \hat{\mathbf{E}}_n^H \mathbf{a}(z). \quad (2.70)$$

The polynomial $p(z)$ has $2K$ roots with K roots inside the unit circle and K mirrored images outside the unit circle. Among those inside, the phases of the K closest to the unit circle gives the AOA estimates.

The Root-MUSIC has empirically been found to have a significant better performance than MUSIC in the small sample case according to [28, 71].

- Coherent Signals

For highly correlated or coherent signals, a rank deficiency occurs in the source covariance matrix $\mathbf{R}_s$. This results in a divergence of a signal eigenvector into the noise subspace. Thus the property (2.59) no longer holds and the subspace method fails to yield consistent estimates. The forward-backward (FB) averaging and spatial smoothing techniques can be used with subspace methods for ULAs in the limiting case of coherent signals [6, 72, 44, 73]. The idea of the FB averaging is to form a so-called backward array covariance matrix $\mathbf{R}_B$ and then average the usual array covariance $\mathbf{R}$ and $\mathbf{R}_B$. The backward array covariance matrix has the form

$$\mathbf{R}_B = \mathbf{J}\mathbf{R}^*\mathbf{J} = \mathbf{A}\mathbf{\Phi}^{-(L-1)}\mathbf{R}_s\mathbf{\Phi}^{-(L-1)}\mathbf{A}^H + \sigma^2\mathbf{I}, \quad (2.71)$$

where $\mathbf{J}$ is an $L \times L$ permutation matrix, whose components are zero excepts for ones on the anti-diagonal. Then the FB array covariance matrix is obtained by

$$\mathbf{R}_{FB} = \frac{1}{2}(\mathbf{R} + \mathbf{R}_B) = \mathbf{A}\widetilde{\mathbf{R}}_s\mathbf{A}^H + \sigma^2\mathbf{I}, \quad (2.72)$$

where $\widetilde{\mathbf{R}}_s = (\mathbf{R}_s + \mathbf{\Phi}^{-(L-1)}\mathbf{R}_s\mathbf{\Phi}^{-(L-1)})/2$ has full rank.

The idea of the spatial smoothing technique is to split the ULA into a number of overlapping subarrays. The outputs of the subarrays can therefore be averaged for computing the covariance matrix. Since the spatial smoothing induces a random phase modulation similar to (2.72), the signals are decorrelated and results in a full rank covariance matrix.

Subspace-Based ML Approximations

Subspace-based methods offer significant performance improvement in comparison to conventional beamforming methods. Although the MUSIC method yields estimates with the same large sample accuracy as that of the deterministic ML method provided the signals are uncorrelated [20], it usually suffer from a large bias for finite samples and is unable to cope with coherent signals. Several parametric subspace-based methods that practically have the same statistical performance as the ML method but less computational complexity than ML have been proposed [25, 26, 23, 63].

• Signal Subspace Fitting

Recall the relation in (2.64). Since $\boldsymbol{\theta}$ and $\mathbf{T}$ are unknown, it appears there are no values of $\boldsymbol{\theta}$ such that $\mathbf{E}_s = \mathbf{A}\mathbf{T}$ when only an estimate $\hat{\mathbf{E}}_s$ of $\mathbf{E}_s$ is available. A natural solution is to find the best weighted least-squares fit of the two subspaces by the following criterion:

$$\{\hat{\boldsymbol{\theta}}, \hat{\mathbf{T}}\} = \arg \min_{\boldsymbol{\theta}, \mathbf{T}} \|\hat{\mathbf{E}}_s - \mathbf{A}\mathbf{T}\|_{\mathbf{W}}^2, \quad (2.73)$$

where $\|\mathbf{A}\|_{\mathbf{W}}^2 = \text{tr}\{\mathbf{A}\mathbf{W}\mathbf{A}^H\}$ and $\mathbf{W}$ is a $K' \times K'$ positive definite weighting matrix. This is a separable nonlinear least-squares problem [74]. Substituting the pseudo-inverse solution, $\hat{\mathbf{T}} = \mathbf{A}^\dagger \hat{\mathbf{E}}_s$, into (2.73) leads to the separated criterion function

$$\hat{\boldsymbol{\theta}}_{SSF} = \arg \min_{\boldsymbol{\theta}} \text{Tr}\{\mathbf{P}_{\mathbf{A}}^\perp \hat{\mathbf{E}}_s \mathbf{W} \hat{\mathbf{E}}_s^H\}. \quad (2.74)$$

Different choices of $\mathbf{W}$ lead to different methods. The choice $\mathbf{W} = \mathbf{I}$ yields Cadzow's method [75]. A statistical analysis suggests other weightings, such as

$$\hat{\mathbf{W}} = (\hat{\mathbf{\Lambda}}_s - \hat{\sigma}^2 \mathbf{I})^2 \hat{\mathbf{\Lambda}}_s^{-1}, \quad (2.75)$$

leading to superior performance [25, 23, 76]. The estimator defined by (2.74) with weights given by (2.75) is called the MODE/WSF (Weighted Subspace Fitting) method.

For a ULA, the MODE algorithm [25, 26] can be implemented efficiently. The idea is to reparameterize the projection matrix $\mathbf{P}_{\mathbf{A}}^\perp$ using a basis for the nullspace of $\mathbf{A}^H$. The projector $\mathbf{P}_{\mathbf{A}}^\perp$ above may be reparameterized in terms of the coefficients $\mathbf{b} = [b_0 \ b_1 \ \dots \ b_K]^T$ of a polynomial defined as

$$\sum_{k=0}^K b_k z^{K-k} = b_0 \prod_{k=1}^K (z - e^{j\Omega_k}); \quad b_0 \neq 0. \quad (2.76)$$

Let $\mathbf{B}^H$ be the following $(L - K) \times L$ matrix

$$\mathbf{B}^H = \begin{bmatrix} b_K & \cdots & b_1 & b_0 & 0 \\ & \ddots & & \ddots & \ddots \\ 0 & b_K & \cdots & b_1 & b_0 \end{bmatrix}. \quad (2.77)$$

Then $\mathbf{P}_{\mathbf{A}}^\perp = \mathbf{P}_{\mathbf{B}}$, where $\mathbf{P}_{\mathbf{B}}$ denotes the orthogonal projector onto the range space of $\mathbf{B}$. Thus the reparameterized SSF criterion for MODE is given by

$$\hat{\mathbf{b}} = \arg \min_{\mathbf{b}} \text{Tr}\{\mathbf{B}(\mathbf{B}^H \mathbf{B})^{-1} \mathbf{B}^H \hat{\mathbf{E}}_s \mathbf{W} \hat{\mathbf{E}}_s^H\}. \quad (2.78)$$

Clearly, the resulting non-iterative procedure includes:

- 1) Determine $\mathbf{b}$ by minimizing (2.78) with $(\mathbf{B}^H \mathbf{B})^{-1}$ set to $\mathbf{I}$;
- 2) Determine an asymptotically efficient estimate $\hat{\mathbf{b}}$ as

$$\hat{\mathbf{b}} = \arg \min_{\mathbf{b}} \text{Tr}\{\mathbf{B}(\hat{\mathbf{B}}^H \hat{\mathbf{B}})^{-1} \mathbf{B}^H \hat{\mathbf{E}}_s \mathbf{W} \hat{\mathbf{E}}_s^H\}, \quad (2.79)$$

where $(\hat{\mathbf{B}}^H \hat{\mathbf{B}})$ is formed by the initial estimate of $\mathbf{b}$ obtained in Step 1;

- 3) Compute the roots of the estimated polynomial.

This MODE algorithm for a ULA is attractive because of the fact that it is computationally and asymptotically efficient, and it remains so for highly correlated or coherent signals. Two-dimensional spatial frequency and angle estimation using MODE has been presented in [46, 77].

• Noise Subspace Fitting

Inspired by MUSIC, if signal covariance $\mathbf{R}_s$ has full rank, then the following relation holds:

$$\mathbf{E}_n^H \mathbf{A} = 0. \quad (2.80)$$

Given an estimate of $\mathbf{E}_n$, a natural estimate of $\boldsymbol{\theta}$ can be obtained by minimizing the following Noise Subspace Fitting (NSF) criterion:

$$\hat{\boldsymbol{\theta}}_{NSF} = \arg \min_{\boldsymbol{\theta}} \text{tr}\{\mathbf{U} \mathbf{A}^H \hat{\mathbf{E}}_n \hat{\mathbf{E}}_n^H \mathbf{A}\}, \quad (2.81)$$

where $\mathbf{U}$ is a $K \times K$ positive semi-definite weighting matrix. Different choices of weighting matrix $\mathbf{U}$ result in signal parameter estimates with different asymptotic

properties. It has been shown in [76] that the estimates calculated by (2.81) and (2.74) are asymptotically equivalent if the weighting matrix is given by

$$\mathbf{U} = \mathbf{A}^\dagger \hat{\mathbf{E}}_s \mathbf{W} \hat{\mathbf{E}}_s^H \mathbf{A}^{\dagger H}. \quad (2.82)$$

Compared with SSF, however, NSF has some inherent disadvantages, namely that it may not cope with coherent signals, and a two-step procedure has to be adopted because the weighting matrix depends on $\boldsymbol{\theta}$.

The parametric high resolution algorithms introduced in this subsection are very sensitive to the presence of unknown spatially colored noise. The performance of these algorithms is usually very poor when used in practical applications. To derive more robust algorithms, we may need to relax the spatially white noise assumption made by these algorithms. A robust RELAX algorithm will be proposed in Chapter 3 for angle and waveform estimation of narrowband plane waves arriving at a ULA. The RELAX algorithm does not assume that the noise is spatially white.

2.5 Wideband Signals

The methods introduced above are essentially limited to narrowband signals. For wideband signals, the delay spread is close to $1/BW$, where BW is the bandwidth of signals. The wideband signal is usually decomposed into narrowband components by filtering or DFT (Discrete Fourier Transform), and then somehow combine the different frequency bins before using the narrowband algorithms. The question of how to best combine the information spread across different frequencies has received much

attention. The algorithms proposed in [78, 79] process each frequency bin separately by using MUSIC, and then averages the spatial spectrums, whereas [80] proposes to use ESPRIT. These methods are incoherent in the sense that the directional information is spread over a wideband of frequencies, and each frequency is processed separately followed by some statistical average of the results. This average of the results tends to be ineffective at low SNR and thus the resolution of these methods tends to be limited by the resolution of a single frequency bin. A number of focusing algorithms have been proposed including coherent signal subspace methods (CSM) [29, 30], and the spatial resampling method [81]. These coherent methods are an alternative to the incoherent methods. These methods preprocess the data, focusing the different frequency components to a signal reference frequency, removing the frequency dependence of the data. Other wideband AOA estimation algorithms have been developed in [82, 83].

We now briefly review the wideband array data model and the concept of CSM. Assume that K incident wideband signals impinge at a uniform linear array with L sensors. The data model in frequency domain has the form

$$\mathbf{y}(f_n) = \mathbf{A}(\boldsymbol{\theta}, f_n)\mathbf{\Gamma}(f_n) + \mathbf{N}(f_n), \quad n = 1, 2, \dots, J, \quad (2.83)$$

where $\mathbf{y}(f_n)$ denotes the $L \times 1$ data vector of the n -th frequency bin, $\mathbf{A}(\boldsymbol{\theta}, f_n) = [\mathbf{a}(\theta_1, f_n) \ \dots \ \mathbf{a}(\theta_K, f_n)]$ is a $L \times K$ matrix, whose k -th column is the direction

vector for the k -th signal, and $\mathbf{\Gamma}(f_n) = [\Gamma_1(f_n) \ \dots \ \Gamma_K(f_n)]^T$, is the $K \times 1$ signal vector, and $\mathbf{N}(f_n) = [N_1(f_n) \ \dots \ N_L(f_n)]^T$, is the $L \times 1$ noise vector.

The resulting spatial covariance matrix is

$$\mathbf{R}(f_n) = \mathbf{A}(\boldsymbol{\theta}, f_n) \mathbf{R}_s(f_n) \mathbf{A}^H(\boldsymbol{\theta}, f_n) + \sigma_n^2(f_n) \mathbf{R}_n(f_n), \quad (2.84)$$

where $\mathbf{R}_s(f_n)$ and $\mathbf{R}_n(f_n)$ are the signal covariance matrix and the noise covariance matrix, respectively. The most important question here is how to combine the information from different frequency bins. The idea of the coherent signal-subspace method (CSM) [29, 30] is to use transformation matrices to focus the energy of the wideband signals to a single reference frequency. The focusing removes the frequency dependency of the data. These focusing transformation matrices $\mathbf{T}(f_n)$ are designed to align the signal-subspace at all frequencies with the signal subspace at the reference frequency f_0 ,

$$\mathbf{T}(f_n) \mathbf{A}(\boldsymbol{\theta}, f_n) \approx \mathbf{A}(\boldsymbol{\theta}, f_0), \quad n = 1, \dots, J. \quad (2.85)$$

Thus, a coherent averaging of the covariance matrices $\mathbf{R}(f_n)$, $n = 1, \dots, J$, may be performed as below

$$\begin{aligned} \mathbf{R}_y &= \frac{1}{J} \sum_{n=1}^J \mathbf{T}(f_n) \mathbf{R}(f_n) \mathbf{T}^H(f_n) \\ &\approx \mathbf{A}(\boldsymbol{\theta}, f_0) \mathbf{R}_s \mathbf{A}^H(\boldsymbol{\theta}, f_0) + \mathbf{R}_{ny} \end{aligned} \quad (2.86)$$

where

$$\mathbf{R}_s = \frac{1}{J} \sum_{n=1}^J \mathbf{R}_s(f_n), \quad (2.87)$$

and

$$\mathbf{R}_{ny} = \frac{1}{J} \sum_{n=1}^J \sigma_n^2(f_n) \mathbf{T}(f_n) \mathbf{R}_n(f_n) \mathbf{T}^H(f_n). \quad (2.88)$$

Hence any narrowband subspace-based algorithms may be applied to $\hat{\mathbf{R}}_y$ after the coherent averaging is performed.

It is apparent from (2.85) that the transformation matrix depends on the unknown AOAs $\boldsymbol{\theta}$. To form the focusing transformation matrices, one needs to have preliminary estimates of AOAs. The preliminary estimates may be obtained by other methods, such as, FFT or Capon's method [29, 30]. A major disadvantage of CSM is that the focusing may cause biased estimates, depending on the accuracy of the preliminary estimates. We will proposed a wideband RELAX algorithm for angle estimation of wideband plane waves arriving at a uniform linear array. The wideband RELAX does not need initial estimates as other wideband algorithms require and naturally focuses the narrowband components in the spatial frequency domain. Thus the wideband RELAX can avoid the bias problem encountered by CSM that uses focusing matrices.

2.6 Polarization Diversity

We have considered an array of omnidirectional isotropic sensors so far. When a polarization sensitive array is used, the output of sensor array is related to the states of polarization of the incident plane waves. In other words, $a_l(\theta_k)$ in (2.26) is

no longer constant, it depends on the polarization property of antenna sensors and incident EM plane waves. The advantages of utilizing polarization sensitive arrays have been discussed previously [10, 84, 85, 31, 86, 87, 33, 65, 88, 89, 90, 91]. In [10, 84] the MUSIC algorithm was used for direction finding with a diversely polarized antenna array. In [92, 31, 86, 87] the methods of angle and polarization estimation using ESPRIT with a crossed dipole array were presented. A noise subspace fitting method with diversely polarized antenna arrays was proposed in [65]. The discussion on the performance analysis of diversely polarized antenna array can be found in [88, 89]. A ML parameter estimator with a crossed dipole array was proposed in [93] for partially polarized EM waves. However, there are many shortcomings in the previous work. For example, the noise subspace fitting and MUSIC cannot work well for coherent signals, the performance of the crossed dipole array is sensitive to AOA's. New approaches will be proposed in Chapters 5 and 6 to overcome these problems. We will consider the case where all incident narrowband EM plane waves are completely polarized.

2.7 Other Applications

The research progress of parameter estimation algorithms in sensor array processing has resulted in a great diversity of applications, such as multiuser estimation and detection, channel identification and spatial diversity in personal communications.

The MUSIC algorithm was used in [94] for propagation delay estimation of the DS-CDMA (Direct-Sequence Code Division Multiple Access) signals. In [95, 96] a

large sample maximum likelihood (LSML) algorithm was proposed for propagation delay, carrier phase, amplitude estimation of DS-CDMA signals. These algorithms have been shown to be robust against the multiuser near-far problem of DS-CDMA systems. Array signal processing algorithms are expected to play an important role in accommodating a multiuser communication environment, subject to severe multipath.

2.8 Summary

We have reviewed important properties of electromagnetic waves including wave propagation and wave polarization. The previous work in sensor array processing has been reviewed for both nonparametric and parametric methods. We have also described the relations of our work herein with the previous work. A rather complete list of references on sensor array processing can be found in [97].

CHAPTER 3

NARROWBAND ANGLE AND WAVEFORM ESTIMATION VIA RELAX

3.1 Introduction

Many high resolution array processing algorithms have been devised to estimate the incident angles of the signals arriving at an array of sensors. The success of those algorithms is typically demonstrated by means of simulated numerical examples, often under the additive white Gaussian noise assumption. The high resolution angle estimation algorithms, however, are usually very sensitive to the violation of the data model assumptions. As a result, the performance of those algorithms is frequently much worse when they are used in practical applications. To derive more robust array processing algorithms, we shall, for instance, relax the additive white noise assumption.

In this chapter we describe how the RELAX algorithm, recently proposed in [45] for mixed spectrum estimation, can be used for angle and waveform estimation of narrowband plane waves arriving at a uniform linear array in the presence of spatially colored noise, such as an autoregressive (AR) or autoregressive moving average (ARMA) noise. The RELAX algorithm presented in this paper is devised for the multisnapshot case. To evaluate the performance of the RELAX algorithm, we apply RELAX to both simulated and experimental data, and compare the performance of RELAX

with that of other well-known algorithms including ESPRIT with forward/backward spatial smoothing [12, 72], MODE/WSF [25, 23], and AP/ANPA [66, 67] under the same scenarios. The experimental data was collected by the Multi-parameter Adaptive Radar System (MARS) developed at McMaster University. More importantly, we explain why RELAX outperforms other well-known algorithms including ESPRIT, MODE/WSF, and AP/ANPA. For the experimental data, since we do not know the true incident angles, we introduce a cross-validatory method to assess the quality of the estimates obtained with the algorithms we consider.

3.2 Problem Formulation

Consider the problem of angle and waveform estimation of K narrowband plane waves impinging on a uniform linear array (ULA) with M ($M \geq 2K$) elements. Assume the number of incident plane waves K is already known. The array output vector can be written as:

$$\mathbf{y}(n) = \mathbf{A}(\boldsymbol{\theta})\mathbf{s}(n) + \mathbf{e}(n), \quad n = 1, 2, \dots, N. \quad (3.1)$$

In (3.1), N is the number of temporal snapshots, $\mathbf{e}(n)$ is a complex $M \times 1$ noise vector, $\mathbf{s}(n)$ is the complex $K \times 1$ signal vector, and

$$\mathbf{A}(\boldsymbol{\theta}) = \begin{bmatrix} \mathbf{a}(\theta_1) & \mathbf{a}(\theta_2) & \dots & \mathbf{a}(\theta_K) \end{bmatrix}, \quad (3.2)$$

where $\boldsymbol{\theta} = \begin{bmatrix} \theta_1 & \theta_2 & \dots & \theta_K \end{bmatrix}^T$ and $\mathbf{a}(\theta_k)$, $k = 1, 2, \dots, K$, is the complex $M \times 1$ direction vector for the k th signal arriving from angle θ_k relative to the array normal.

(Here $(\cdot)^T$ denotes the transpose.) For the ULA, the k th direction vector $\mathbf{a}(\theta_k)$ has the form:

$$\mathbf{a}(\theta_k) = \begin{bmatrix} 1 & e^{j\frac{2\pi\delta}{\lambda} \sin \theta_k} & \dots & e^{j\frac{2\pi\delta}{\lambda} (M-1) \sin \theta_k} \end{bmatrix}^T, \quad k = 1, 2, \dots, K, \quad (3.3)$$

where δ is the spacing between the array sensors and λ is the signal wavelength.

The noise vector $\mathbf{e}(n)$ is a spatially correlated random process that is assumed to be temporally white. In particular, $\mathbf{e}(n)$ has zero-mean and unknown covariance matrix $\mathbf{Q}$:

$$\mathbb{E}\{\mathbf{e}(n_1)\mathbf{e}^H(n_2)\} = \mathbf{Q}\delta_{n_1, n_2}, \quad (3.4)$$

where $(\cdot)^H$ denotes the complex conjugate transpose and δ_{n_1, n_2} is the Kronecker delta.

The signal waveforms $\mathbf{s}(n) = \begin{bmatrix} s_1(n) & s_2(n) & \dots & s_K(n) \end{bmatrix}^T$, $n = 1, 2, \dots, N$, are modeled as deterministic unknowns. This assumption is usually referred to as the deterministic (or conditional) signal model [20, 76].

The problem of interest herein is to estimate the angles $\theta_1, \theta_2, \dots, \theta_K$ and the complex waveforms $s_1(n), s_2(n), \dots, s_K(n)$ from $\mathbf{y}(n)$, $n = 1, 2, \dots, N$.

3.3 Angle and Waveform Estimation Using RELAX

The RELAX algorithm was derived in [45] as an estimator of sinusoidal parameters in the presence of colored noise. We describe below how the RELAX algorithm can be modified for angle and waveform estimation with a uniform linear array of sensors.

The angle and waveform estimates $\{\hat{\theta}, \hat{s}(1), \hat{s}(2), \dots, \hat{s}(N)\}$ can be obtained via RELAX by minimizing, with respect to θ and $s(1), s(2), \dots, s(N)$, the following nonlinear least-squares (NLS) criterion:

$$F_1[\theta, s(1), s(2), \dots, s(N)] = \sum_{n=1}^N [y(n) - \mathbf{A}(\theta)s(n)]^H [y(n) - \mathbf{A}(\theta)s(n)]. \quad (3.5)$$

Note that if the noise $\mathbf{e}(n)$ were spatially white and Gaussian, minimizing (3.5) would have yielded the deterministic maximum likelihood estimates of the incident angles and signal waveforms [28, 66].

We present below our approach to the minimization of the cost function in (3.5). Assume for now that there are $\bar{K}$ signals, where $\bar{K}$ is an intermediate value ($\bar{K} \leq K$, see the steps below). Let

$$\mathbf{y}_k(n) = \mathbf{y}(n) - \sum_{i=1, i \neq k}^{\bar{K}} \mathbf{a}(\hat{\theta}_i) \hat{s}_i(n), \quad (3.6)$$

where $\{\hat{\theta}_i, \hat{s}_i(1), \hat{s}_i(2), \dots, \hat{s}_i(N)\}_{i=1, i \neq k}^{\bar{K}}$ are assumed to have been previously estimated. Then the cost function for estimating the k th signal parameters becomes

$$F_2(\theta_k, s_k(1), s_k(2), \dots, s_k(N)) = \sum_{n=1}^N [\mathbf{y}_k(n) - \mathbf{a}(\theta_k)s_k(n)]^H [\mathbf{y}_k(n) - \mathbf{a}(\theta_k)s_k(n)], \quad (3.7)$$

The minimization of F_2 with respect to θ_k and $s_k(1), \dots, s_k(N)$ gives

$$\hat{s}_k(n) = \frac{\mathbf{a}^H(\theta_k) \mathbf{y}_k(n)}{M} \Big|_{\theta_k = \hat{\theta}_k}, \quad n = 1, 2, \dots, N, \quad (3.8)$$

and

$$\hat{\theta}_k = \arg \min_{\theta_k} \sum_{n=1}^N \left\| \mathbf{I} - \frac{\mathbf{a}(\theta_k) \mathbf{a}^H(\theta_k)}{M} \right\| \mathbf{y}_k(n) \|^2 = \arg \max_{\theta_k} \sum_{n=1}^N |\mathbf{a}^H(\theta_k) \mathbf{y}_k(n)|^2. \quad (3.9)$$

Hence $\hat{\theta}_k$ is obtained as the location of the dominant peak of the sum of the periodograms $|\mathbf{a}^H(\theta_k) \mathbf{y}_k(n)|^2 / M$, over $n = 1, 2, \dots, N$, which can be efficiently computed by using FFT (fast Fourier transform) with each of the data sequence $\mathbf{y}_k(n)$ padded with zeros. (Note that padding with zeros is necessary to determine $\hat{\theta}_k$ with high accuracy.) Furthermore, $\hat{s}_k(n)$ is easily computed from the complex height of the peak of $\mathbf{a}^H(\theta_k) \mathbf{y}_k(n) / M$ (for $n = 1, 2, \dots, N$).

We can now proceed to present our relaxation (RELAX) algorithm for the minimization of the nonlinear least-squares cost function in (3.5). Note that we assume the number of incident signals K is already known or estimated. The RELAX algorithm comprises the following steps:

Step (1): Let $\bar{K} = 1$. Obtain $\hat{\theta}_1$ and $\hat{s}_1(n)$ from $\mathbf{y}_1(n)$ as in (3.8) and (3.9),
 $n = 1, 2, \dots, N$.

Step (2): Let $\bar{K} = 2$. Compute $\mathbf{y}_2(n)$ with (3.6) by using $\hat{\theta}_1$ and $\hat{s}_1(n)$ obtained in Step (1), $n = 1, 2, \dots, N$. Obtain $\hat{\theta}_2$ and $\hat{s}_2(n)$ from $\mathbf{y}_2(n)$ as in (3.8) and (3.9),
 $n = 1, 2, \dots, N$.

Next, compute $\mathbf{y}_1(n)$ with (3.6) by using $\hat{\theta}_2$ and $\hat{s}_2(n)$ and redetermine $\hat{\theta}_1$ and $\hat{s}_1(n)$ from $\mathbf{y}_1(n)$ as in (3.8) and (3.9), $n = 1, 2, \dots, N$.

Iterate the previous two substeps until “practical convergence” is achieved (to be discussed later on).

Step (3): Let $\bar{K} = 3$. Compute $\mathbf{y}_3(n)$ with (3.6) by using $\{\hat{\theta}_i, \hat{s}_i(1), \hat{s}_i(2), \dots, \hat{s}_i(N)\}_{i=1}^2$ obtained in Step (2). Obtain $\hat{\theta}_3$ and $\hat{s}_3(n)$ from $\mathbf{y}_3(n)$ as in (3.8) and (3.9), $n = 1, 2, \dots, N$.

Next, compute $\mathbf{y}_1(n)$ with (3.6) by using $\{\hat{\theta}_i, \hat{s}_i(1), \hat{s}_i(2), \dots, \hat{s}_i(N)\}_{i=2}^3$ and re-determine $\hat{\theta}_1$ and $\hat{s}_1(n)$ from $\mathbf{y}_1(n)$, $n = 1, 2, \dots, N$. Then compute $\mathbf{y}_2(n)$ with (3.6) by using

$\{\hat{\theta}_i, \hat{s}_i(1), \hat{s}_i(2), \dots, \hat{s}_i(N)\}_{i=1,3}$ and redetermine $\hat{\theta}_2$ and $\hat{s}_2(n)$ from $\mathbf{y}_2(n)$, $n = 1, 2, \dots, N$.

Iterate the previous three substeps until “practical convergence” is achieved.

Remaining Steps: Continue similarly until $\bar{K}$ is equal to the known number K or the estimated number $\hat{K}$ of signal sources. (See [45] for a possible approach to the estimation of K when it is unknown.)

The “practical convergence” in the iterated substeps of the above RELAX algorithm may be determined by checking the relative change of the cost function in (3.5) between two consecutive iterations. In our simulated and experimental examples, to be presented in the next subsection, we terminate the iterations when the relative change is less than or equal to $\epsilon = 10^{-3}$, i.e.,

$$\frac{|F_1[\hat{\theta}_{q+1}, \hat{s}_{q+1}(1), \dots, \hat{s}_{q+1}(N)] - F_1[\hat{\theta}_q, \hat{s}_q(1), \dots, \hat{s}_q(N)]|}{F_1[\hat{\theta}_q, \hat{s}_q(1), \dots, \hat{s}_q(N)]} \leq \epsilon, \quad (3.10)$$

where $\hat{\theta}_q, \hat{s}_q(1), \dots, \hat{s}_q(N)$ denote the estimates obtained after the q th iteration.

To show how RELAX works, we consider an example shown in Figure 3.1, where $N = 1$, $K = 2$, $\theta_1 = 35.5^\circ$, $\theta_2 = 38.5^\circ$, $s_1(1) = 1$, $s_2(1) = 1$, and the ULA has $M = 32$ sensors with the spacing between two adjacent sensors equal to a half wavelength of the incident plane waves. The frequency axis used in the figure is related to AOs by $f_k = \frac{1}{2} \sin(\theta_k)$, $k = 1, 2$, where f_1 and f_2 are shown with the dashed line. The FFTs of $\mathbf{y}_k$ in Figure 3.1 is obtained when the array data are zero padded to 1024. Note that the AOA separation 3° is less than the FFT resolution 3.6° . However, the AOs can be estimated accurately with RELAX after a few of iterations.

3.4 Results of Simulated and Experimental Data

We first present a simulation example for estimating the angles and waveforms of incident signals. The simulation results were obtained by using 50 Monte-Carlo trials. The noise $e_m(n)$ is assumed to be a spatial AR random process of order 1, i.e.,

$$e_m(n) = -a_1 e_{m-1}(n) + w_m(n), \quad (3.11)$$

where $a_1 = -0.85$ and $w_m(n)$, $m = 1, 2, \dots, M$, are independently and identically distributed zero-mean complex Gaussian random variables with variance σ^2 . The complex amplitudes of the waveforms are assumed to be 1. The SNR referred to below is defined by [98]

$$\text{SNR} = 10 \log_{10} \left(\frac{1 - |a_1|^2}{\sigma^2} \right) \text{ [dB]}. \quad (3.12)$$

We assume that there are $K = 2$ correlated incident signals, with the correlation coefficient equal to 0.99, arriving from angles $\theta_1 = -10^\circ$ and $\theta_2 = 2^\circ$. The number of incident signals is assumed known. The array is assumed to have $M = 8$ sensors that are uniformly spaced with the spacing between two adjacent sensors equal to a half wavelength. The number of snapshots taken at the array output is $N = 10$. All array output vectors are zero padded to 2^{13} before using them with FFT in the RELAX and ANPA algorithms. (Note that the higher the SNR, the more padded zeros are needed so that the estimation accuracy is not affected by the FFT intervals.) The waveform estimates for MODE, ANPA, and LS-ESPRIT (least squares based ESPRIT) [12] are obtained by using

$$\hat{\mathbf{s}}(n) = [\mathbf{A}^H(\hat{\boldsymbol{\theta}})\mathbf{A}(\hat{\boldsymbol{\theta}})]^{-1}\mathbf{A}^H(\hat{\boldsymbol{\theta}})\mathbf{y}(n). \quad (3.13)$$

The root-mean-squared error (RMSE) of the k th waveform estimate is defined as $\sqrt{\sum_{n=1}^N |s_k(n) - \hat{s}_k(n)|^2}$. Note that LS-ESPRIT is used with forward/backward spatial smoothing and subarray length equal to 4 (that is, $M/2$) due to the highly correlated signals.

Figures 3.2(a) and (b) show the RMSEs of the angle and waveform estimates, respectively, for the first signal as a function of SNR. (The results for the second signal are similar.) The estimator performances are also compared to the corresponding Cramér-Rao bound (CRB). (The CRB matrix for the signal parameters is derived in Appendix A.) Note that RELAX has the best performance among the four algorithms for both angle and waveform estimation, especially for waveform estimation at low SNR. RELAX outperforms MODE and ESPRIT since the noise is not spatially white

as the latter methods assume. ANPA gives bad waveform estimates at low SNR because using ANPA may yield almost identical angle estimates in some Monte-Carlo trials. The ANPA angle estimates are obtained by maximizing the following cost function:

$$F_3(\boldsymbol{\theta}) = \sum_{n=1}^N \|\mathbf{P}_{\mathbf{A}(\boldsymbol{\theta})} \mathbf{y}(n)\|^2, \quad (3.14)$$

which can be obtained by first minimizing the cost function in (3.5) with respect to $\mathbf{s}(1), \mathbf{s}(2), \dots, \mathbf{s}(N)$ [66, 67], where $\mathbf{P}_{\mathbf{A}} = \mathbf{A} (\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H$. Although using (3.13) for waveform estimation assumes that the angle estimates are not nearly identical, maximizing $F_3(\boldsymbol{\theta})$ does not guarantee this assumption. Figure 3.3 shows $F_3(\boldsymbol{\theta})$ as a function of θ_1 and θ_2 when the realization of the noise is the one that gives the worst waveform estimates for ANPA in Figure 1. It can be seen from Figure 3.3 that the maximum of the cost function corresponds to almost identical angles. Hence maximizing the cost function in (3.14) with respect to $\boldsymbol{\theta}$ (only) and using the estimated $\hat{\boldsymbol{\theta}}$ to estimate $\mathbf{s}(n)$ with (3.13), instead of leading to a simpler problem, can actually complicate the optimization of the cost function in (3.5) and lead to poorer estimates when the incident angles are closely spaced.

We next apply the algorithms to the experimental data collected by the array system known as the Multi-parameter Adaptive Radar System (MARS) [99]. The array system was developed at the Communications Research Laboratory at McMaster University. The data was collected by deploying the array at the west coast of the Bruce Peninsula, Ontario, Canada, overlooking Lake Huron. MARS is a vertical uniform linear array consisting of $M = 32$ horizontally polarized horn antennas. The

spacing between adjacent antenna sensors is 5.715 cm. The four sets of data we use below were collected when the array system was operated at frequencies 9.76, 9.79, 11.32 and 12.34 GHz. The data was recorded with 12-bit precision and sampled at 62.5 samples per second. For each carrier frequency, 127 snapshots were collected at each antenna output. There are two incident signals, a fact assumed to be known to the angle estimation algorithms. One of the incident signals is the direct path and the other is the specular path, which is reflected from the lake. The direct incident signal is a continuous wave (CW), whose amplitude is a constant and whose phase is a linear function of time. Since the specular path is a delayed and reflected version of the direct path, the phase difference between the two paths is a constant, which is determined by the time delay between the direct and specular paths, the carrier frequency of the waves, and the reflection coefficient of the lake. The incident signals arrive from near the array normal, but the exact incident angles are unknown since the vertical array structure may have been on a slight tilt. The parameter estimation algorithms we consider below do not assume any *a priori* knowledge of the incident angles and the signal waveforms.

In this application we use a cross-validatory criterion to assess the quality of estimates. Consider the case of where the algorithms are applied to a single snapshot at a time. Let $\hat{\theta}_s$ be the estimated angle vector obtained from the s th snapshot by using one of the algorithms. Then our cross-validatory criterion is:

$$C = \frac{1}{\bar{N}(\bar{N} - 1)} \sum_{s=1}^{\bar{N}} \sum_{n=1, n \neq s}^{\bar{N}} \|\mathbf{P}_{\mathbf{A}(\hat{\theta}_s)}^\perp \mathbf{y}(n)\|^2, \quad (3.15)$$

where $\bar{N}$ is the number of those snapshots that do not give almost identical angle estimates, and $\mathbf{P}_A^\perp = \mathbf{I} - \mathbf{A} (\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H$. (Note that almost identical angle estimates cause the matrix $\mathbf{A}^H \mathbf{A}$ in $\mathbf{P}_{\mathbf{A}(\hat{\boldsymbol{\theta}}_s)}^\perp$ to be ill conditioned.) What the criterion does here may be explained by the following steps: 1) estimate AOAs from a single snapshot; 2) obtain least squares estimate of waveforms by using the estimate of AOAs for every other snapshot, i.e., $\hat{\mathbf{s}}(n) = [\mathbf{A}^H(\hat{\boldsymbol{\theta}}_s) \mathbf{A}(\hat{\boldsymbol{\theta}})]^{-1} \mathbf{A}^H(\hat{\boldsymbol{\theta}}_s) \mathbf{y}(n)$, ($n = 1, \dots, N$, and $n \neq s$); 3) obtain the residue by subtracting the signal part from the data with the data model for each snapshot, i.e., $residue(n) = \mathbf{y}(n) - \mathbf{A}(\hat{\boldsymbol{\theta}}_s) \hat{\mathbf{s}}(n) = \mathbf{P}_{\mathbf{A}(\hat{\boldsymbol{\theta}}_s)}^\perp \mathbf{y}(n)$, ($n = 1, \dots, N$, and $n \neq s$); 4) find the summation of residues for each of these snapshots as in (3.15). Clearly, the smaller the summation, the better the performance of the algorithm. The inner sum in (3.15) shows how well (or bad) the estimate obtained from snapshot s can be used to predict the array outputs observed in the other available snapshots, according to the assumed data model. The outer sum adds those prediction errors together. The lower the C , the better the performance of the algorithm.

Table 1 shows the cross-validatory criterion C obtained by using RELAX, ANPA, MODE, LS-ESPRIT, and TLS-ESPRIT (total least squares based ESPRIT) [12] at frequencies 8.62, 9.76, 9.79, and 12.34 GHz. Note that LS-ESPRIT and TLS-ESPRIT are used with forward/backward spatial smoothing and subarray length equal to 10. In Table 1, the failure rate indicates how often an algorithm gives almost identical angle estimates, and hence very poor waveform estimates. Note that for the data set analyzed here, RELAX and ANPA have similar performances and are better than the eigendecomposition based MODE and ESPRIT, which assume that the additive

noise is spatially white (note that the noise statistics are unknown). To see whether the noise is actually spatially correlated, we estimate the noise covariance matrix as follows:

$$\hat{\mathbf{Q}} = \frac{1}{(N-1)N} \sum_{s=1}^{\bar{N}} \sum_{n=1, n \neq s}^N \mathbf{P}_{\mathbf{A}(\hat{\boldsymbol{\theta}}_s)}^{\perp} \mathbf{y}(n) \mathbf{y}^H(n) \mathbf{P}_{\mathbf{A}(\hat{\boldsymbol{\theta}}_s)}^{\perp}. \quad (3.16)$$

Figure 3.4 shows the correlation coefficients between the first sensor noise and the other sensor noises. The noise appears to be strongly spatially correlated, which explains the poorer performance of the eigenstructure based methods. Finally, we note from Table 1 that LS-ESPRIT and TLS-ESPRIT have similar performances for this data set, but TLS-ESPRIT has a much higher failure rate, which indicates that when waveform estimation is desired, LS-ESPRIT should be preferred over TLS-ESPRIT.

Finally, we compare the computational complexities of these algorithms. The amount of computations needed by RELAX depends on N , K , SNR and the number of zero-paddings. For example, RELAX requires about 40% of the amount of computations required by ANPA when $N = 1$, $K = 2$, SNR = 0 dB and the number of zero-paddings is 2^{12} . Since ESPRIT and MODE do not need iterations, they require much fewer computations than RELAX and ANPA. For example, ESPRIT and MODE only need about 0.3% and 1.5%, respectively, of the amount of computations required by RELAX when $N = 1$, $K = 2$, SNR = 0 dB and the number of zero-paddings is 2^{12} . However, RELAX can still be attractive because it can be easily implemented with simple FFT chips in parallel.

3.5 Conclusions

We have presented a RELAX algorithm for the multi-snapshot case of angle and waveform estimation of narrowband plane waves arriving at a uniform linear array. The RELAX algorithms are conceptually and computationally simple; their implementations mainly require a sequence of fast Fourier transforms. We have evaluated the performance of the RELAX algorithm by using both simulated and experimental data, and compared it with the performances of other well-known algorithms such as ESPRIT with forward/backward spatial smoothing, MODE/WSF, and AP/ANPA. We have explained by means of results of both simulated and experimental data why better signal parameter estimates can be obtained by using RELAX as compared to using the other algorithms.

Table 1: Cross-validatory criterion for the different algorithms when used with the experimental data collected by MARS.

Algorithms		8.62 GHz	9.76 GHz	9.79 GHz	12.34 GHz
RELAX	Criterion C	1.3578e+04	2.9153e+04	6.6079e+04	1.7177e+04
RELAX	Failure rate	0	0	0	0
ANPA	Criterion C	1.3323e+04	2.9209e+04	6.6260e+04	1.7172e+04
ANPA	Failure rate	0	0	0	0
MODE	Criterion C	4.5306e+04	3.8840e+04	7.2059e+04	2.6091e+04
MODE	Failure rate	38%	9%	5%	6%
LS- ESPRIT	Criterion C	1.5353e+04	3.3716e+04	11.0460e+04	1.8302e+04
LS- ESPRIT	Failure rate	0	0	0	0
TLS- ESPRIT	Criterion C	1.6723e+04	3.3712e+04	11.4600e+04	1.8302e+04
TLS- ESPRIT	Failure rate	47%	0	9%	0

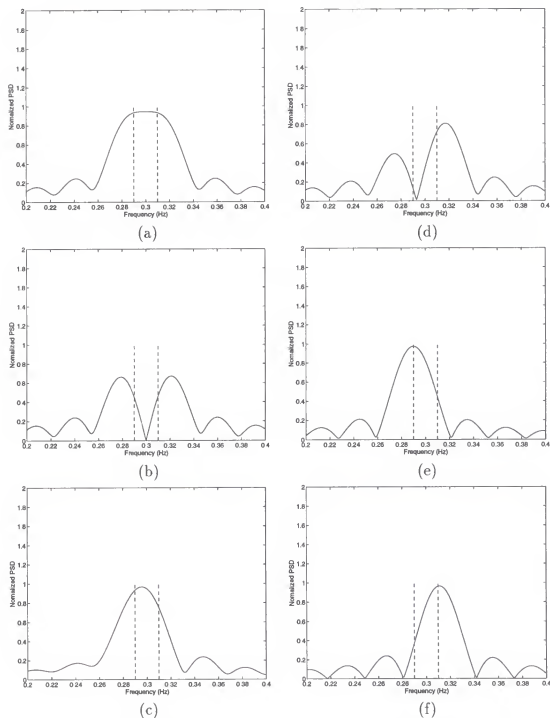
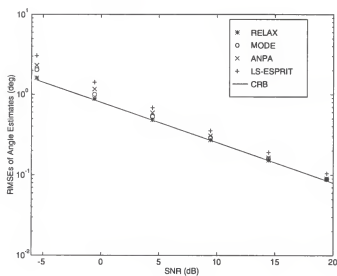
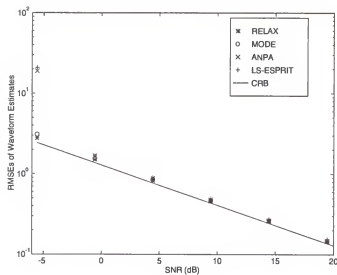


Figure 3.1. An example of using RELAX. (a) Modulus of FFT of y in step (1), $\bar{K} = 1$. (b) Modulus of FFT of y_2 in step (2), $\bar{K} = 2$ (1st iteration). (c) Modulus of FFT of y_1 in step (2), $\bar{K} = 2$ (2nd iteration). (d) Modulus of FFT of y_2 in step (2), $\bar{K} = 2$ (3rd iteration). (e) Modulus of FFT of y_1 in step (2), $\bar{K} = 2$ (4th iteration). (f) Modulus of FFT of y_2 in step (2), $\bar{K} = 2$ (5th iteration).

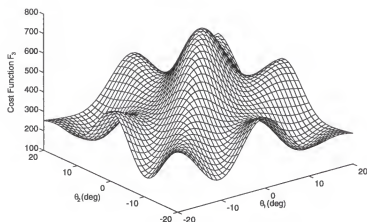


(a)

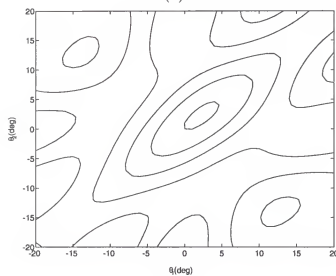


(b)

Figure 3.2. RMSEs of the angle and waveform estimates of the first signal as a function of SNR when $N = 10$, $M = 8$, and $K = 2$ correlated incident signals with the correlation coefficient equal to 0.99 arrive from $\theta_1 = -10^\circ$ and $\theta_2 = 2^\circ$. (a) Angle estimation. (b) Waveform estimation.



(a)



(b)

Figure 3.3. F_3 vs. θ_1 and θ_2 when $N = 10$, $M = 8$, the correlation coefficient of the two incident signals is equal to 0.99, and the realization of the noise is the one that gives the worst waveform estimates for ANPA in Figure 1. (a) Mesh plot. (b) Contour plot.

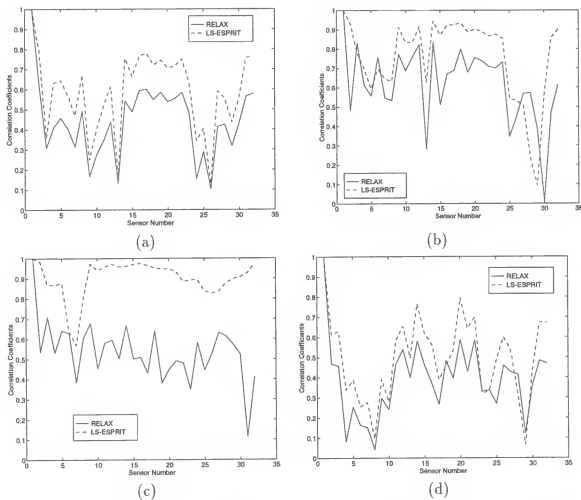


Figure 3.4. Estimated noise correlation coefficients between the first and the other sensors. Figures (a) – (d) are for the carrier frequencies 8.62, 9.76, 9.79, and 12.34 GHz, respectively.

CHAPTER 4 ANGLE ESTIMATION OF WIDEBAND SIGNALS USING RELAX

4.1 Introduction

In this chapter we extend the RELAX algorithm to the case of *wideband* sources. We concentrate on angle estimation herein. We show that the wideband RELAX algorithm we devise naturally focuses the narrowband components in the spatial frequency domain. We use both numerical and experimental examples to demonstrate the performance of the wideband RELAX algorithm and compare its performance with that of the well-known CSM [29] based LS-ESPRIT (CSM-ESPRIT). We also explain why wideband RELAX can outperform the CSM-ESPRIT.

4.2 Problem Formulation

Consider the same uniform linear array (ULA) with M elements as before. Assume that the incident wideband deterministic signals have a common bandwidth B (Hz) with center frequency f_0 (Hz). The k th bandpass signal $s_k(t)$ observed at a reference point can be written as

$$s_k(t) = \gamma_k(t)e^{j2\pi f_0 t}, \quad (4.1)$$

where $\gamma_k(t)$ denotes the complex envelope. Let the signal be observed over a duration $[t_0, t_0 + T_0]$. The complex envelope can be written as

$$\gamma_k(t) = \sum_{l=1}^L \Gamma_k(f_l) e^{j2\pi f_l t}, \quad t_0 \leq t \leq t_0 + T_0, \quad (4.2)$$

where

$$\Gamma_k(f_l) = \frac{1}{T_0} \int_{t_0}^{t_0+T_0} \gamma_k(t) e^{-j2\pi f_l t} dt, \quad (4.3)$$

with $f_l = \frac{l-(L+1)/2}{T_0}$, $l = 1, 2, \dots, L$. L is the number of frequency components symmetrically placed around 0 Hz with $f_{l+1} - f_l = \frac{B}{L-1} = \frac{1}{T_0}$. Thus the bandpass signal at the reference point (say the first sensor) can be written as

$$s_k(t) = \sum_{l=1}^L \Gamma_k(f_l) e^{j2\pi(f_0+f_l)t}. \quad (4.4)$$

The k th signal at the m th sensor has a propagation time delay $\tau_{k,m}$ so that

$$s_k(t + \tau_{k,m}) = \sum_{l=1}^L \Gamma_k(f_l) e^{j2\pi(f_0+f_l)(t+\tau_{k,m})} \triangleq x_k(t, \tau_{k,m}) e^{j2\pi f_0 t}, \quad (4.5)$$

where

$$\tau_{k,m} = \frac{(m-1)\delta}{C} \sin \theta_k, \quad k = 1, 2, \dots, K, \quad m = 1, 2, \dots, M, \quad (4.6)$$

with δ being the spacing between sensors, C the propagation speed, and θ_k the angle of arrival of the k th incident signal relative to the array normal.

Let $\mathbf{x}_k(t) = \begin{bmatrix} x_k(t, \tau_{k,1}) & x_k(t, \tau_{k,2}) & \cdots & x_k(t, \tau_{k,M}) \end{bmatrix}^T$. Then

$$\mathbf{x}_k(t) = \sum_{l=1}^L \mathbf{a}(\theta_k, f_0 + f_l) \Gamma_k(f_l) e^{j2\pi f_l t}, \quad (4.7)$$

where $\mathbf{a}(\theta_k, f_0 + f_l)$ is the $M \times 1$ direction vector of the k th source and has the form

$$\mathbf{a}(\theta_k, f_0 + f_l) = \begin{bmatrix} e^{j2\pi(f_0+f_l)\tau_{k,1}} & e^{j2\pi(f_0+f_l)\tau_{k,2}} & \cdots & e^{j2\pi(f_0+f_l)\tau_{k,M}} \end{bmatrix}^T. \quad (4.8)$$

Hence if K wideband signals along with some noise simultaneously impinge upon the sensor array, the received data vector has the form

$$\mathbf{x}(t) = \sum_{l=1}^L [\mathbf{A}(\boldsymbol{\theta}, f_0 + f_l) \boldsymbol{\Gamma}(l) + \mathbf{N}(l)] e^{j2\pi f_l t} = \sum_{l=1}^L \mathbf{y}(l) e^{j2\pi f_l t}, \quad (4.9)$$

where

$$\mathbf{A}(\boldsymbol{\theta}, f_0 + f_l) = \begin{bmatrix} \mathbf{a}(\theta_1, f_0 + f_l) & \cdots & \mathbf{a}(\theta_K, f_0 + f_l) \end{bmatrix}, \quad (4.10)$$

and

$$\boldsymbol{\Gamma}(l) = \begin{bmatrix} \Gamma_1(f_l) & \cdots & \Gamma_K(f_l) \end{bmatrix}^T, \quad (4.11)$$

is the $K \times 1$ signal vector, and

$$\mathbf{N}(l) = \begin{bmatrix} N_1(f_l) & \cdots & N_M(f_l) \end{bmatrix}^T, \quad (4.12)$$

is the $M \times 1$ noise Fourier coefficient vector, and $\{\mathbf{y}(l)\}$ are by definition the $M \times 1$ Fourier coefficient vectors of $\mathbf{x}(t)$:

$$\mathbf{y}(l) = \mathbf{A}(\boldsymbol{\theta}, f_0 + f_l)\boldsymbol{\Gamma}(l) + \mathbf{N}(l), \quad l = 1, 2, \dots, L. \quad (4.13)$$

We assume that the noise vector $\mathbf{N}(l)$ has zero-mean and

$$\mathbb{E}\{\mathbf{N}(l_1)\mathbf{N}^H(l_2)\} = \mathbf{Q}\delta_{l_1, l_2}, \quad (4.14)$$

where $\mathbf{Q}$ is unknown.

The problem of interest herein is to estimate $\theta_1, \theta_2, \dots, \theta_K$ from $\mathbf{y}(l)$, $l = 1, 2, \dots, L$. The main difference with respect to the narrowband problem treated in Section 2 is that now the array transfer matrix $\mathbf{A}$ depends on the snapshot index l .

4.3 Angle Estimation Using RELAX

The estimates $\{\hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\Gamma}}(1), \dots, \hat{\boldsymbol{\Gamma}}(L)\}$ can be obtained via RELAX by minimizing the following nonlinear least-square criterion with respect to $\boldsymbol{\theta}$ and $\boldsymbol{\Gamma}(1), \dots, \boldsymbol{\Gamma}(L)$ (See (3.5)):

$$G_1 = \sum_{l=1}^L [\mathbf{y}(l) - \mathbf{A}(\boldsymbol{\theta}, f_0 + f_l)\boldsymbol{\Gamma}(l)]^H [\mathbf{y}(l) - \mathbf{A}(\boldsymbol{\theta}, f_0 + f_l)\boldsymbol{\Gamma}(l)]. \quad (4.15)$$

To estimate the parameters of the k th signal, consider

$$G_2 = \sum_{l=1}^L [\mathbf{y}_k(l) - \mathbf{a}(\theta_k, f_0 + f_l))\Gamma_k(l)]^H [\mathbf{y}_k(l) - \mathbf{a}(\theta_k, f_0 + f_l))\Gamma_k(l)], \quad (4.16)$$

where

$$\mathbf{y}_k(l) = \mathbf{y}(l) - \sum_{i=1, i \neq k}^{\bar{K}} \mathbf{a}(\hat{\theta}_i, f_0 + f_l)\hat{\Gamma}_i(l), \quad (4.17)$$

and where $\bar{K}$ is as defined before, and $\{\hat{\theta}_i, \hat{\Gamma}_i(1), \dots, \hat{\Gamma}_i(L)\}_{i=1, i \neq k}^{\bar{K}}$ are assumed to be given or estimated. Let

$$Y_k(l) = \frac{\mathbf{a}^H(\theta_k, f_0 + f_l)\mathbf{y}_k(l)}{M}, \quad (4.18)$$

which can be obtained by using FFTs with zero padding. Then similar to the narrowband RELAX algorithm, $\hat{\theta}_k$ can be obtained by

$$\hat{\theta}_k = \arg \max_{\theta_k} \sum_{l=1}^L |Y_k(l)|^2, \quad (4.19)$$

and

$$\hat{\Gamma}_k(l) = Y_k(l)|_{\theta_k = \hat{\theta}_k}, \quad l = 1, 2, \dots, L. \quad (4.20)$$

Hence $\hat{\theta}_k$ is obtained from the location of the dominant peak of the sum of the focused periodograms $|Y_k(l)|^2$, $l = 1, 2, \dots, L$. Then $\hat{\Gamma}_k(l)$ is easily computed from the complex height of the peak of $Y_k(l)$. Note that the focusing of the narrowband components can be naturally achieved in (4.19) by expanding and compressing the FFTs (with zero padding) of $\mathbf{y}_k(l)$ according to the ratio $(f_0 + f_l)/f_0$ since the spacing

between two adjacent FFT samples are different for different l . The steps of our RELAX algorithm for minimizing (6.20) are summarized as follows:

Step (0): Obtain $\mathbf{y}(l)$ from $\mathbf{x}(t)$ via DFT (discrete Fourier transform).

Step (1): Assume $\bar{K} = 1$. Obtain $\hat{\theta}_1$ and $\hat{\Gamma}_1(l)$ from $\mathbf{y}(l)$ as in (4.19) and (4.20),
 $l = 1, 2, \dots, L$.

Step (2): Assume $\bar{K} = 2$. Compute $\mathbf{y}_2(l)$ with (4.17) by using $\hat{\theta}_1$ and $\hat{\Gamma}_1(l)$ obtained in Step (1), $l = 1, 2, \dots, L$. Obtain $\hat{\theta}_2$ and $\hat{\Gamma}_2(l)$ from $\mathbf{y}_2(l)$ as in (4.19) and (4.20), $l = 1, 2, \dots, L$.

Next, compute $\mathbf{y}_1(l)$ with (4.17) by using $\hat{\theta}_2$ and $\hat{\Gamma}_2(l)$ and redetermine $\hat{\theta}_1$ and $\hat{\Gamma}_1(l)$ from $\mathbf{y}_1(l)$, $l = 1, 2, \dots, L$.

Iterate the previous two substeps until “practical convergence” is achieved. (See Section 2 for details.)

Remaining Steps: Continue similarly until $\bar{K}$ is equal to the known number K or the estimated number $\hat{K}$ of signals.

We refer to this algorithm as the *wideband RELAX*.

4.4 Numerical Results

We present below numerical and experimental examples showing the performance of the proposed algorithm for estimating the incident angles of wideband signals in the presence of either white or unknown colored noise. We also compare the performance of wideband RELAX with that of CSM based LS-ESPRIT (CSM-ESPRIT) for angle estimation.

In both of the simulation examples below, the array is assumed to be a ULA of $M = 8$ sensors with the spacing between two adjacent sensors equal to half of the wavelength corresponding to the center frequency f_0 . The wideband sources have the same center frequency $f_0 = 100$ Hz and the same bandwidth $B = 40$ Hz. The noise is a temporally stationary zero-mean white Gaussian process, independent of the signals, and spatially either a white or an autoregressive (AR) process. The total observation time is $T_0 = 0.8$ seconds. The demodulated data is sampled at twice the Nyquist rate. The array output is decomposed into $L = 33$ narrowband components via the DFT (discrete Fourier transform). The signal-to-noise ratio (SNR) is defined as the ratio of the power of each signal to the noise power. We assume that there are $K = 2$ independent wideband signals with equal power impinging on the array from $\theta_1 = 10^\circ$ and $\theta_2 = 20^\circ$. Note that the two signals cannot be resolved by the spatial periodogram. All sequences are zero padded to 4096 for FFT in the RELAX algorithm. The simulation results were obtained by using 30 Monte-Carlo simulations.

Figure 4.1(a) shows the root-mean-squared error (RMSE) of the second signal as a function of SNR in the presence of white noise. (The results for the first signal are similar.) We note that the RMSE of the wideband RELAX decreases with SNR, but the RMSE of CSM-ESPRIT stops decreasing when SNR reaches a certain level. This is because, due to focusing, CSM-ESPRIT provides biased angle estimates even when the angle separation of the two sources is within the resolution of the spatial periodogram.

In Figure 4.1(b), we consider the performance of both wideband RELAX and CSM-ESPRIT in the presence of unknown AR noise. The noise $N_m(f_i)$ is assumed to be a complex AR process of order 1, i.e, $N_m(f_i) = -a_1 N_{m-1}(f_i) + W_m(f_i)$, where $a_1 = -0.85e^{-j\frac{\pi}{4}}$ and $W_m(f_i)$ is a zero-mean complex white Gaussian random process with variance equal to σ^2 . The SNR is defined by $-10 \log_{10} \frac{\sigma^2}{1-|a_1|^2}$ dB. Note that wideband RELAX again performs better than CSM-ESPRIT.

Finally, we apply both RELAX and CSM-ESPRIT to the experimental data collected by the sensor array testbed [100, 101] at the University of Minnesota. The uniform linear array consists of 8 sensors. The spacing between adjacent sensors is about 2.1 times the wavelength corresponding to f_0 . Two correlated sources arrive from around -33° and -36° . The SNR is 21 dB for each source. The center carrier frequency for this data is 40 kHz. The bandwidth of the data is 4 kHz. The data was sampled at a rate of 5 kHz, and was decomposed into $L = 5$ frequency bins (38.125 kHz, 39.0625 kHz, 40 kHz, 40.9375 kHz, and 41.875 kHz). Figure 4.2 shows the angle estimates obtained from several observation intervals. (Note that one of the angle estimates does not show up in Figure 4.2 (b) because it is too small.) The means and the standard deviations of the angle estimates in Figure 4.2 were calculated by averaging the angle estimates obtained from all 64 observation intervals. Note that wideband RELAX provides smaller standard deviations and biases than CSM-ESPRIT.

4.5 Conclusions

A wideband RELAX algorithm for the angle estimation of wideband sources has been presented. The wideband RELAX naturally focuses the narrowband components in the spatial frequency domain. Numerical and experimental examples have shown that the wideband RELAX can perform better than CSM.

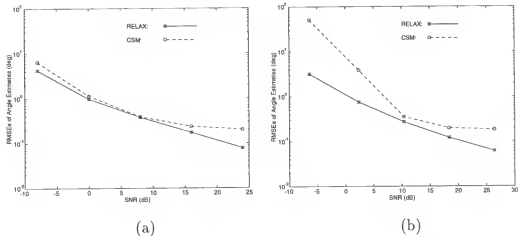


Figure 4.1. RMSEs of the angle estimates of the second signal as a function of SNR when $K = 2$ uncorrelated wideband signals arrive from $\theta_1 = 10^\circ$ and $\theta_2 = 20^\circ$, $M = 8$, and $L = 33$. (a) In the presence of white noise. (b) In the presence of unknown AR noise.

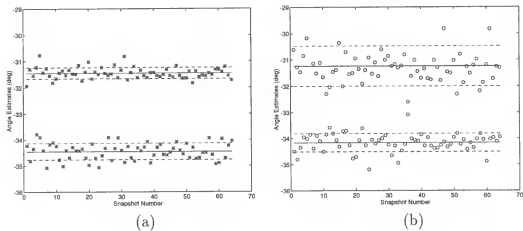


Figure 4.2. Angle estimates obtained from the experimental data, corresponding to 64 observation intervals. The solid lines denote the means and the dashed lines denote the means plus and minus the standard deviations of the angle estimates. The true incident angles are believed to be $\theta_1 = -33^\circ$ and $\theta_2 = -36^\circ$. (a) RELAX. (b) CSM-ESPRIT.

CHAPTER 5

ANGLE AND POLARIZATION ESTIMATION WITH A COLD ARRAY

5.1 Introduction

This chapter studies the advantages of an arbitrary linear array that consists of Co-centered Orthogonal Loop and Dipole (COLD) pairs. By using the COLD array, the performance of both angle and polarization estimation can be significantly improved as compared to using a co-centered crossed dipole (CCD) array.

The case where all incident narrowband electromagnetic (EM) plane waves are completely polarized is considered. A completely polarized EM wave is a limiting case of a more general type of EM wave, viz. a partially polarized EM wave. The state of polarization of a partially polarized EM wave is a function of time while a completely polarized wave has a fixed state of polarization (see [102] and the references therein).

We present an asymptotically statistically efficient signal subspace-based MODE algorithm [25, 26] for both angle and polarization estimation. Since the MODE algorithm is a signal subspace-based approach, it is asymptotically statistically efficient for both correlated (including coherent) and uncorrelated incident signals. We show with numerical examples that the estimation performance of MODE is better, especially for highly correlated or coherent signals, than that of MUSIC and NSF (noise subspace fitting) [65]. (We remark that the signal subspace eigenvector based MODE

algorithm and the NSF algorithm are asymptotically statistically equivalent whenever the signals are non-coherent. For coherent signals, MODE remains asymptotically statistically efficient, whereas NSF is no longer asymptotically statistically efficient [76]. This observation suggests that when the correlation coefficient is close/very close to one, the NSF may need a much larger number of data samples than MODE to converge to the asymptotics, and hence for a given finite N , MODE is likely to perform better than NSF in such a case of highly correlated signals.)

5.2 COLD Array and Problem Formulation

Consider a $2L$ -element linear array consisting of L Co-centered Orthogonal Loop and Dipole (COLD) pairs as shown in Figure 5.1. The signal received from each antenna sensor is to be processed separately for direction and polarization estimation. The l th COLD pair, $l = 1, 2, \dots, L$, has its center on the y -axis at an arbitrary $y = \delta_l$. For the l th COLD pair, the dipole parallel to the z -axis is referred to as the z -axis dipole and the loop parallel to the x - y plane as the $x - y$ plane loop.

Assume K (with $K \leq L$) narrowband plane waves impinge on the array from angular directions described by θ and ϕ , where θ and ϕ denote the azimuth and elevation angles, respectively, as shown in Figure 5.1. Furthermore, suppose each signal is a completely polarized transverse electromagnetic wave with an arbitrary elliptical electromagnetic polarization [103]. Assume that the electric field of an incoming signal has transverse components

$$\mathbf{E} = E_\theta \mathbf{e}_\theta + E_\phi \mathbf{e}_\phi, \quad (5.1)$$

where unit vectors $\mathbf{e}_\theta$, $\mathbf{e}_\phi$, and $-\mathbf{e}_r$, in that order, form a right-hand coordinate system for the incoming signals and E_θ and E_ϕ are the horizontal and vertical components of the electric field respectively. In general, as time progresses, E_θ and E_ϕ will describe a polarization ellipse. For a given signal polarization, specified by constants γ and η , the electric field components are given by (aside from a common narrowband phase factor $s_o(t)$ ¹)

$$E_\theta = E \cos \gamma, \quad (5.2)$$

$$E_\phi = E \sin \gamma e^{j\eta}, \quad (5.3)$$

where E denotes the amplitude of the incident signal. The γ and η can be used to compute α and β , which are the ellipticity and orientation angles of the polarization ellipse, respectively. γ is always in the range $0 \leq \gamma \leq \pi/2$ and η is in the range $-\pi \leq \eta < \pi$. α and β can also be used to compute γ and η [31, 40].

Assume that each dipole in the array is a short dipole (i.e., the length of the dipole is equal to or less than one-tenth of a wavelength) with the same length L_{sd} and each loop is a small loop (i.e., the perimeter of the loop is equal to or less than three-tenths of a wavelength) with the same area A_{sl} . Thus the output voltages from each dipole and loop are proportional to the electric field components parallel to dipole and loop, respectively.

¹For a narrowband BPSK (binary phase-shift keyed) signal, for example, $s_o(t) = e^{j[\omega_o t + \psi(t)]}$, where ω_o is the carrier frequency and $\psi(t)$ is the modulating phase.

An incoming signal described by arbitrary electric field components E_θ and E_ϕ can be written as

$$\mathbf{E} = E \left[(\cos \gamma) \mathbf{e}_\theta + (\sin \gamma e^{j\eta}) \mathbf{e}_\phi \right]. \quad (5.4)$$

Let us define the spatial phase factor

$$q_l = e^{j \frac{2\pi d_l}{\lambda_0} \sin \theta}, \quad (5.5)$$

where λ_0 is the wavelength of the signal. The effective heights of the short dipoles and small loops are given by [104]

$$h_{sd} = L_{sd} \sin \phi, \quad (5.6)$$

and

$$h_{sl} = -j \frac{2\pi A_{sl}}{\lambda_0} \sin \phi, \quad (5.7)$$

respectively. Including the time and space phase factors in (5.4), we find that an incoming signal characterized by $(\theta, \phi, \gamma, \eta, E)$ produces a signal vector in the COLD pair centered at $y = \delta_l$ as follows:

$$\mathbf{z}_l(t) = \begin{bmatrix} x_l(t) \\ x_d(t) \end{bmatrix} = \boldsymbol{\mu}_{COLD} E s_o(t) q_l, \quad (5.8)$$

where

$$\boldsymbol{\mu}_{COLD} = \begin{bmatrix} j \frac{2\pi A_{sl}}{\lambda_0} \sin \phi \cos \gamma \\ -L_{sd} \sin \phi \sin \gamma e^{j\eta} \end{bmatrix}. \quad (5.9)$$

An advantage of the COLD array is that its antenna elements are not sensitive to the azimuth angle θ of the signal because both the loops and dipoles have the same $\sin \phi$ field pattern, as may be seen from Equation (5.9). Hence the incoming signal described by (5.4) is independent of θ . We assume that the antennas and the incident signals are co-planar, i.e., $\phi = 90^\circ$. Thus (5.9) becomes

$$\boldsymbol{\mu}_{COLD} = \begin{bmatrix} V_\theta \cos \gamma \\ V_\phi \sin \gamma e^{j\eta} \end{bmatrix}, \quad (5.10)$$

where

$$V_\theta = j \frac{2\pi A_{sl}}{\lambda_0}, \quad (5.11)$$

$$V_\phi = -L_{sd}. \quad (5.12)$$

Note that V_θ and V_ϕ represent the complex voltages induced at the loop and dipole outputs by a signal with a unit electric field parallel to the loops and dipoles, respectively. Let $s(t) = E s_o(t) V_\theta \cos \gamma$. The $\mathbf{z}_l(t)$ in (5.8) can be rewritten as

$$\mathbf{z}(t) = \mathbf{u} s(t) \mathbf{q}_l, \quad (5.13)$$

where

$$\mathbf{u} = \begin{bmatrix} 1 \\ \frac{V_\theta}{V_\theta} \tan \gamma e^{j\eta} \end{bmatrix} \equiv \begin{bmatrix} 1 \\ r \end{bmatrix}. \quad (5.14)$$

Assume that K signals, specified by incident angles θ_k , $k = 1, 2, \dots, K$, are incident on the array. In addition, we assume a thermal noise voltage vector $\mathbf{n}_l(t)$ is present at each output vector $\mathbf{z}_l(t)$. The $\mathbf{n}_l(t)$ are assumed to be zero-mean circularly symmetric complex Gaussian random processes that are statistically independent of each other and to have covariance matrix $\sigma^2 \mathbf{I}$, where $\mathbf{I}$ denotes the identity matrix.

Under these assumptions, the total output vector received by the COLD pair centered at $y = \delta_l$ is given by

$$\mathbf{z}_l(t) = \sum_{k=1}^K \mathbf{u}_k s_k(t) q_{lk} + \mathbf{n}_l(t), \quad l = 1, 2, \dots, L, \quad (5.15)$$

where $\mathbf{u}_k$ and q_{lk} are given by (6.9) and (5.5), respectively, with subscript k added to each angular quantity. Further, $s_k(t) = E_k s_{ok}(t) V_\theta \cos \gamma_k$, where $E_k s_{ok}(t)$ denotes the k th narrowband signal. The incident signals may or may not be correlated (including completely correlated, i.e., coherent) with each other.

Let $\mathbf{z}(t)$, $\mathbf{s}(t)$, and $\mathbf{n}(t)$ be column vectors containing the received signals, incident signals, and noise, respectively, i.e.,

$$\mathbf{z}(t) = \begin{bmatrix} \mathbf{z}_1(t) \\ \mathbf{z}_2(t) \\ \vdots \\ \mathbf{z}_L(t) \end{bmatrix}, \quad \mathbf{s}(t) = \begin{bmatrix} s_1(t) \\ s_2(t) \\ \vdots \\ s_K(t) \end{bmatrix}, \quad \mathbf{n}(t) = \begin{bmatrix} \mathbf{n}_1(t) \\ \mathbf{n}_2(t) \\ \vdots \\ \mathbf{n}_L(t) \end{bmatrix}. \quad (5.16)$$

The received signal vector has the form

$$\mathbf{z}(t) = \overline{\mathbf{A}}\mathbf{s}(t) + \mathbf{n}(t), \quad (5.17)$$

where $\overline{\mathbf{A}}$ is a $2L \times K$ matrix

$$\begin{aligned} \overline{\mathbf{A}} &= \begin{bmatrix} \overline{\mathbf{a}}_1 & \overline{\mathbf{a}}_2 & \cdots & \overline{\mathbf{a}}_K \end{bmatrix} \\ &= [\mathbf{A} \otimes \mathbf{I}]\mathbf{U}, \end{aligned} \quad (5.18)$$

with $\otimes$ representing the Kronecker product,

$$\mathbf{A} = \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{1K} \\ q_{21} & q_{22} & \cdots & q_{2K} \\ \vdots & \vdots & & \vdots \\ q_{L1} & q_{L2} & \cdots & q_{LK} \end{bmatrix}, \quad (5.19)$$

and

$$\mathbf{U} = \begin{bmatrix} \mathbf{u}_1 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{u}_K \end{bmatrix}. \quad (5.20)$$

Assume that the element signals are sampled at N distinct times t_n , $n = 1, 2, \dots, N$. The random noise vectors $\mathbf{n}(t_n)$ at different sample times are assumed to be independent of each other. The problem of interest herein is to determine the azimuth arrival angles θ_k and the states of polarization described by (γ_k, η_k) or (α_k, β_k) , $k = 1, 2, \dots, K$, from the measurements $\mathbf{z}(t_n)$, $n = 1, 2, \dots, N$.

5.3 Angle and Polarization Estimation using MODE

The MODE [25, 26] and, in a related form, the WSF [23] algorithms were derived for angle estimation with uniformly polarized arrays. We present below how to use the signal subspace-based *MODE* algorithm with the COLD array for both angle and polarization estimation.

Let

$$\hat{\mathbf{R}} = \frac{1}{N} \sum_{n=1}^N \mathbf{z}(t_n) \mathbf{z}^H(t_n), \quad (5.21)$$

where $(\cdot)^H$ denotes the complex conjugate transpose and $\hat{\mathbf{R}}$ denotes the estimate of the following array covariance matrix:

$$\mathbf{R} = E[\mathbf{z}(t) \mathbf{z}^H(t)]. \quad (5.22)$$

It has been shown in [25, 23, 76] that an asymptotically (for large N) statistically efficient estimator of the angles $\boldsymbol{\theta} = [\theta_1, \theta_2, \dots, \theta_K]^T$ and the polarization parameters $\mathbf{r} = [r_1, r_2, \dots, r_K]^T$ can be obtained by minimizing the following function:

$$f(\boldsymbol{\theta}, \mathbf{r}) = \text{Tr} \left[\mathbf{P}_{\hat{\mathbf{A}}}^\perp \hat{\mathbf{E}}_s \hat{\hat{\mathbf{A}}}^2 \hat{\mathbf{A}}^{-1} \hat{\mathbf{E}}_s^H \right], \quad (5.23)$$

where the symbol $\mathbf{P}_{\hat{\mathbf{A}}}^\perp$ stands for the orthogonal projector onto the null space of $\overline{\mathbf{A}}^H$, and the columns in $\hat{\mathbf{E}}_s$ are the signal subspace eigenvectors of $\hat{\mathbf{R}}$ that correspond to the $\tilde{K}$ largest eigenvalues of $\hat{\mathbf{R}}$, with $\tilde{K} = \min[N, \text{rank}(\mathbf{S})]$. Here $\mathbf{S}$ is the source covariance matrix,

$$\mathbf{S} = E\{\mathbf{s}(t_n)\mathbf{s}^H(t_n)\}. \quad (5.24)$$

Assume that $\tilde{K}$ is known. (If $\tilde{K}$ is unknown, it can be estimated from the data as described, for example, in [43].) Note that if no components of the signal vector $\mathbf{s}(t)$ are fully correlated to one another, then $\tilde{K} = K$ (provided $N > K$). Further, the $\hat{\hat{\mathbf{A}}}$ in (5.23) is a diagonal matrix with diagonal elements $\hat{\lambda}_1 \geq \hat{\lambda}_2 \geq \dots \geq \hat{\lambda}_{\tilde{K}}$, which are the $\tilde{K}$ largest eigenvalues of $\hat{\mathbf{R}}$, and

$$\hat{\hat{\mathbf{A}}} = \hat{\mathbf{A}} - \hat{\sigma}^2 \mathbf{I}, \quad (5.25)$$

where

$$\hat{\sigma}^2 = \frac{1}{2L - \tilde{K}} \sum_{i=\tilde{K}+1}^{2L} \hat{\lambda}_i = \frac{1}{2L - \tilde{K}} \left[\text{tr}(\hat{\mathbf{R}}) - \sum_{i=1}^{\tilde{K}} \hat{\lambda}_i \right]. \quad (5.26)$$

We show below that we can concentrate out $\mathbf{r}$ first and hence reduce the dimension of the parameter space over which we need to search to minimize (5.23).

It has been shown (in Appendix B) that

$$\mathbf{P}_{\hat{\mathbf{A}}}^\perp = \mathbf{I} - \mathbf{P}_{\mathbf{A} \otimes \mathbf{I}} + \mathbf{P}_{(\mathbf{A}^\dagger{}^H \otimes \mathbf{I})\mathbf{V}}, \quad (5.27)$$

where

$$\mathbf{A}^\dagger = (\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H, \quad (5.28)$$

and

$$\mathbf{V} = \begin{bmatrix} \mathbf{v}_1 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{v}_K \end{bmatrix}, \quad (5.29)$$

with

$$\mathbf{v}_k = \begin{bmatrix} -r_k^* \\ 1 \end{bmatrix}, \quad k = 1, 2, \dots, K. \quad (5.30)$$

Thus minimizing $f(\boldsymbol{\theta}, \mathbf{r})$ in (5.23) is equivalent to minimizing

$$\tilde{f}(\boldsymbol{\theta}, \mathbf{r}) = \text{Tr} \left[\left(-\mathbf{P}_{\mathbf{A} \otimes \mathbf{I}} + \mathbf{P}_{(\mathbf{A}^\dagger{}^H \otimes \mathbf{I})\mathbf{V}} \right) \hat{\mathbf{E}}_s \hat{\mathbf{A}}^2 \hat{\mathbf{A}}^{-1} \hat{\mathbf{E}}_s^H \right]. \quad (5.31)$$

Let

$$\mathbf{W} = \left\{ \hat{\mathbf{V}}^H [(\hat{\mathbf{A}}^H \hat{\mathbf{A}})^{-1} \otimes \mathbf{I}] \hat{\mathbf{V}} \right\}^{-1}, \quad (5.32)$$

be formed from some consistent estimates of $\boldsymbol{\theta}$ and $\mathbf{r}$. Since $\mathbf{P}_{(\mathbf{A}^{\dagger H} \otimes \mathbf{I})\mathbf{V}} \hat{\mathbf{E}}_s = O(1/\sqrt{N})$, $\left\{ \mathbf{V}^H [(\mathbf{A}^H \mathbf{A})^{-1} \otimes \mathbf{I}] \mathbf{V} \right\}^{-1}$ can be replaced by $\mathbf{W}$ without affecting the asymptotics of the MODE estimator [25, 26]. Then we have

$$\tilde{f}(\boldsymbol{\theta}, \mathbf{r}) = f_1(\boldsymbol{\theta}) + f_2(\boldsymbol{\theta}, \mathbf{r}), \quad (5.33)$$

where

$$f_1(\boldsymbol{\theta}) = -\text{Tr} \left[(\mathbf{A} \otimes \mathbf{I}) \left\{ (\mathbf{A}^H \mathbf{A})^{-1} \otimes \mathbf{I} \right\} (\mathbf{A}^H \otimes \mathbf{I}) \hat{\mathbf{E}}_s \hat{\mathbf{\Lambda}}^2 \hat{\mathbf{\Lambda}}^{-1} \hat{\mathbf{E}}_s^H \right], \quad (5.34)$$

and

$$f_2(\boldsymbol{\theta}, \mathbf{r}) = \text{Tr} \left[(\mathbf{A}^{\dagger H} \otimes \mathbf{I}) \mathbf{V} \mathbf{W} \mathbf{V}^H (\mathbf{A}^{\dagger} \otimes \mathbf{I}) \hat{\mathbf{E}}_s \hat{\mathbf{\Lambda}}^2 \hat{\mathbf{\Lambda}}^{-1} \hat{\mathbf{E}}_s^H \right]. \quad (5.35)$$

The MODE estimates $\{\hat{\boldsymbol{\theta}}, \hat{\mathbf{r}}\}$ are obtained by minimizing $\tilde{f}$, i.e.,

$$\{\hat{\boldsymbol{\theta}}, \hat{\mathbf{r}}\} = \arg \min_{\boldsymbol{\theta}, \mathbf{r}} [f_1(\boldsymbol{\theta}) + f_2(\boldsymbol{\theta}, \mathbf{r})]. \quad (5.36)$$

Let

$$\mathbf{A}_h = \text{odd columns of } (\mathbf{A}^{\dagger H} \otimes \mathbf{I}), \quad (5.37)$$

and

$$\mathbf{A}_v = \text{even columns of } (\mathbf{A}^{\dagger H} \otimes \mathbf{I}). \quad (5.38)$$

Let $\mathbf{V}_h$ and $\mathbf{V}_v$ be the following $K \times K$ diagonal matrices:

$$\mathbf{V}_h = \text{diag} \left\{ -r_1^*, \dots, -r_K^* \right\}, \quad (5.39)$$

and

$$\mathbf{V}_v = \mathbf{I}. \quad (5.40)$$

Then

$$(\mathbf{A}^{\dagger H} \otimes \mathbf{I})\mathbf{V} = \mathbf{A}_h \mathbf{V}_h + \mathbf{A}_v \mathbf{V}_v. \quad (5.41)$$

Thus $f_2(\boldsymbol{\theta}, \mathbf{r})$ in (5.35) can be rewritten as

$$\begin{aligned} f_2(\boldsymbol{\theta}, \mathbf{r}) = & \text{Tr} \left[\mathbf{V}_h^H \mathbf{A}_h^H \hat{\mathbf{E}}_s \hat{\mathbf{A}}^2 \hat{\mathbf{A}}^{-1} \hat{\mathbf{E}}_s^H \mathbf{A}_h \mathbf{V}_h \mathbf{W} \right] \\ & + \text{Tr} \left[\mathbf{V}_h^H \mathbf{A}_h^H \hat{\mathbf{E}}_s \hat{\mathbf{A}}^2 \hat{\mathbf{A}}^{-1} \hat{\mathbf{E}}_s^H \mathbf{A}_v \mathbf{V}_v \mathbf{W} \right] \\ & + \text{Tr} \left[\mathbf{V}_v^H \mathbf{A}_v^H \hat{\mathbf{E}}_s \hat{\mathbf{A}}^2 \hat{\mathbf{A}}^{-1} \hat{\mathbf{E}}_s^H \mathbf{A}_h \mathbf{V}_h \mathbf{W} \right] \\ & + \text{Tr} \left[\mathbf{V}_v^H \mathbf{A}_v^H \hat{\mathbf{E}}_s \hat{\mathbf{A}}^2 \hat{\mathbf{A}}^{-1} \hat{\mathbf{E}}_s^H \mathbf{A}_v \mathbf{V}_v \mathbf{W} \right]. \end{aligned} \quad (5.42)$$

Since $\mathbf{V}_h$ and $\mathbf{V}_v$ are diagonal matrices, Equation (5.42) can be written in the following matrix form:

$$f_2(\boldsymbol{\theta}, \mathbf{r}) = \begin{bmatrix} \mathbf{v}_h^H & \mathbf{e}^T \end{bmatrix} \mathbf{Q}(\boldsymbol{\theta}) \begin{bmatrix} \mathbf{v}_h \\ \mathbf{e} \end{bmatrix}, \quad (5.43)$$

where

$$\mathbf{Q}(\boldsymbol{\theta}) = \begin{bmatrix} (\mathbf{A}_h^H \hat{\mathbf{E}}_s \hat{\mathbf{A}}^2 \hat{\mathbf{A}}^{-1} \hat{\mathbf{E}}_s^H \mathbf{A}_h) \odot \mathbf{W}^T & (\mathbf{A}_h^H \hat{\mathbf{E}}_s \hat{\mathbf{A}}^2 \hat{\mathbf{A}}^{-1} \hat{\mathbf{E}}_s^H \mathbf{A}_v) \odot \mathbf{W}^T \\ (\mathbf{A}_v^H \hat{\mathbf{E}}_s \hat{\mathbf{A}}^2 \hat{\mathbf{A}}^{-1} \hat{\mathbf{E}}_s^H \mathbf{A}_h) \odot \mathbf{W}^T & (\mathbf{A}_v^H \hat{\mathbf{E}}_s \hat{\mathbf{A}}^2 \hat{\mathbf{A}}^{-1} \hat{\mathbf{E}}_s^H \mathbf{A}_v) \odot \mathbf{W}^T \end{bmatrix} \begin{bmatrix} \mathbf{Q}_1(\boldsymbol{\theta}) & \mathbf{Q}_2(\boldsymbol{\theta}) \\ \mathbf{Q}_2^H(\boldsymbol{\theta}) & \mathbf{Q}_3(\boldsymbol{\theta}) \end{bmatrix}, \quad (5.44)$$

with $\odot$ denoting the Hadamard-Schur matrix product (i.e., the elementwise multiplication),

$$\mathbf{v}_h = \begin{bmatrix} -r_1^* & \cdots & -r_K^* \end{bmatrix}^T, \quad (5.45)$$

and

$$\mathbf{e} = \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}^T. \quad (5.46)$$

Note that the polarization parameters are contained only in $\mathbf{v}_h$. By setting $\partial f_2 / \partial \mathbf{v}_h = \mathbf{0}$, we obtain

$$\hat{\mathbf{v}}_h = -\mathbf{Q}_1^{-1}(\boldsymbol{\theta}) \mathbf{Q}_2(\boldsymbol{\theta}) \mathbf{e}. \quad (5.47)$$

Using (5.47) in (5.43) gives

$$f_3(\boldsymbol{\theta}) = \mathbf{e}^T \left[\mathbf{Q}_3(\boldsymbol{\theta}) - \mathbf{Q}_2^H(\boldsymbol{\theta}) \mathbf{Q}_1^{-1}(\boldsymbol{\theta}) \mathbf{Q}_2(\boldsymbol{\theta}) \right] \mathbf{e}, \quad (5.48)$$

which is a concentrated function depending only on $\boldsymbol{\theta}$. The MODE estimates $\{\hat{\boldsymbol{\theta}}, \hat{\mathbf{r}}\}$ can be obtained by

$$\hat{\boldsymbol{\theta}} = \arg \min_{\boldsymbol{\theta}} [f_1(\boldsymbol{\theta}) + f_3(\boldsymbol{\theta})], \quad (5.49)$$

and using $\hat{\boldsymbol{\theta}}$ in (5.47) to obtain $\hat{\mathbf{r}}$.

To summarize, we have the following MODE algorithm for angle and polarization estimation:

Step 1: Obtain initial estimates of $\boldsymbol{\theta}$ and $\mathbf{r}$ (see the discussions below).

Step 2: Determine $\hat{\boldsymbol{\theta}}$ by minimizing $f_1(\boldsymbol{\theta}) + f_3(\boldsymbol{\theta})$ as shown in Equation (5.49)

with $\mathbf{W}$ in (5.32) formed from the initial estimates obtained in Step 1.

Step 3: Calculate $\hat{\mathbf{r}}$ by using the $\hat{\boldsymbol{\theta}}$ obtained in Step 2 in (5.47).

Step 4: Determine the $\hat{\gamma}$ and $\hat{\eta}$ from $\hat{\mathbf{r}}$ with

$$\hat{\gamma}_k = \tan^{-1} \left(\left| \frac{\hat{r}_k V_{\theta}}{V_{\phi}} \right| \right), \quad (5.50)$$

$$\hat{\eta}_k = -\arg \left(-\frac{\hat{r}_k V_{\theta}}{V_{\phi}} \right), \quad k = 1, 2, \dots, K. \quad (5.51)$$

For signals that are not highly correlated or coherent with each other, the initial estimates of $\boldsymbol{\theta}$ and $\mathbf{r}$ in Step 1 may be obtained by using MUSIC [84], which requires a one-dimensional search over the parameter space. For highly correlated or coherent signals, the initial estimate of $\boldsymbol{\theta}$ may be determined by setting $\mathbf{W} = \mathbf{I}$ and minimizing $f_1(\boldsymbol{\theta}) + f_3(\boldsymbol{\theta})$ as shown in Equation (5.49). The initial estimate of $\mathbf{r}$ can be calculated by using the initial estimate of $\boldsymbol{\theta}$ in (5.47). The initial estimates obtained by using MUSIC for non-coherent signals or MODE with $\mathbf{W} = \mathbf{I}$ are known to be consistent [26, 71].

5.4 Statistical Performance Analysis

We present below the asymptotic (for large N) statistical performance of MODE for both direction and polarization estimation with the COLD array.

Before we present the analysis results, however, we first describe the method we use to describe the accuracy of the polarization estimates. For reasons discussed in [31], we define the polarization estimation error to be the spherical distance between the two points M and $\hat{M}$ on the Poincaré sphere that represent the actual state of polarization (γ, η) and the estimated state of polarization $(\hat{\gamma}, \hat{\eta})$, respectively. Let ζ be the angular distance between M and $\hat{M}$. Then [31]

$$\cos \zeta = \cos 2\gamma \cos 2\hat{\gamma} + \sin 2\gamma \sin 2\hat{\gamma} \cos(\eta - \hat{\eta}). \quad (5.52)$$

where ζ is always in the range $0 \leq \zeta \leq \pi$.

Applying the first-order approximation to the left side of 5.52 yields

$$\zeta_k^2 = 4(\hat{\gamma}_k - \gamma_k)^2 + \sin^2(2\gamma_k)(\hat{\eta}_k - \eta_k)^2. \quad (5.53)$$

The asymptotic variances of the polarization estimates are obtained with (5.53) and the accuracy results on $\hat{\gamma}$ and $\hat{\eta}$ given below.

Let

$$\boldsymbol{\tau} = \begin{bmatrix} \boldsymbol{\theta}^T & \boldsymbol{\gamma}^T & \boldsymbol{\eta}^T \end{bmatrix}^T. \quad (5.54)$$

It follows from [25, 76] that the asymptotic (for large N) statistical distribution of $\hat{\boldsymbol{\tau}}$ is Gaussian with mean $\boldsymbol{\tau}$ and covariance matrix equal to the corresponding stochastic Cramer-Rao bound (CRB), $\mathbf{CRB}$. The ij th element of $\mathbf{CRB}^{-1}$ is given by

$$\left[\mathbf{CRB}^{-1}\right]_{ij} = \frac{2N}{\sigma^2} \text{Re} \left[\text{tr} \left\{ \overline{\mathbf{A}}_j^H \mathbf{P}_{\overline{\mathbf{A}}}^\perp \overline{\mathbf{A}}_i \mathbf{S} \overline{\mathbf{A}}^H \mathbf{R}^{-1} \overline{\mathbf{A}} \mathbf{S} \right\} \right], \quad (5.55)$$

where $\overline{\mathbf{A}}_i = \partial \overline{\mathbf{A}} / \partial \tau_i$ with τ_i denoting the i th element of $\boldsymbol{\tau}$.

5.5 Numerical Results

We present below several examples showing the performance of using the MODE algorithm with the COLD array and comparing the asymptotic statistical performance analysis results with the Monte-Carlo simulation results. We compare MODE with MUSIC and NSF for both angle and polarization estimation. The simulation results were obtained by using 50 Monte-Carlo simulations. In the examples, we assume that there are $K = 2$ incident signals and both signals are assumed to have the same amplitude E_k , such that $|V_\theta E_k| = |V_\phi E_k| = 1$, $k = 1, 2$. Hence, the signal-to-noise ratio (SNR) used in the simulations is $-10 \log_{10} \sigma^2$ dB. The array is assumed to have $L = 8$ COLD pairs that are uniformly spaced with the spacing between two adjacent COLD pairs equal to a half wavelength. We also compare the estimation performance of using the COLD array with that of using a CCD array with the same array geometry. The CCD array consists of crossed y - and z -axes dipoles. The

counterpart of Equation (5.9) for the CCD array can be written as

$$\boldsymbol{\mu}_{CCD} = \begin{bmatrix} L_{sd} \cos \gamma \cos \theta + L_{sd} \sin \gamma e^{j\eta} \sin \theta \cos \phi \\ -L_{sd} \sin \phi \sin \gamma e^{j\eta} \end{bmatrix}. \quad (5.56)$$

In the following examples, the antennas and the incident signals are assumed to be co-planar, i.e., $\phi = 90^\circ$, for both the COLD array and CCD array. For $\phi = 90^\circ$, (5.56) becomes

$$\boldsymbol{\mu}_{CCD} = \begin{bmatrix} L_{sd} \cos \gamma \cos \theta \\ -L_{sd} \sin \gamma e^{j\eta} \end{bmatrix}. \quad (5.57)$$

(We remark that if the antennas and the incident signals are not co-planar, we will need two-dimensional CCD or COLD arrays for angle and polarization estimation, which is the case not considered herein. For this case, however, the COLD array will not always perform better than the CCD array.)

First, we present two examples that illustrate how the angle separation between the two incident signals affects both the direction and polarization estimates. We begin with the case of two signals with identical circular polarizations ($\alpha_1 = \alpha_2 = 45^\circ$). Figure 5.2 shows the root-mean-squared errors (RMSEs) of the estimates of the first signal as a function of angle separation $\Delta\theta$ when two correlated signals with correlation coefficient 0.99 arrive at the array from angles $\theta_1 = -\Delta\theta/2$ and $\theta_2 = \Delta\theta/2$. We note that MODE performs better than MUSIC and NSF. Further, MODE achieves the best possible unbiased performance, i.e., the corresponding CRB, as the angle separation increases. Because the signals arrive from angles near the

broadside of the arrays, the CRBs for the COLD and CCD arrays are similar. This case corresponds to small incident angles, which make μ_{CCD} similar to μ_{COLD} as may be seen by comparing Equations (6.5) and (5.57).

In Figure 5.3, we consider the case where the signals with identical horizontal polarizations ($\alpha_1 = \alpha_2 = 0^\circ, \beta_1 = \beta_2 = 0^\circ$) arrive from angles away from the broadside of the array. In this case, the CCD array is outperformed by the COLD array. This result occurs because the signal outputs at the y -axis dipoles are attenuated by a factor of $\cos \theta$ for the CCD array (see Equation (5.56)). For the COLD array, however, the signal outputs at both the dipoles and the loops are independent of the incident angle θ (see (6.5)). Note also that the RMSEs of the angle estimates first decrease and then increase even as the angle separation increases. This result occurs because the incident angle of the second signal approaches 90° for very large $\Delta\theta$ and the RMSEs of angle estimates are approximately proportional to $\frac{1}{\cos^2 \theta}$ for large $\Delta\theta$ [71]. We note again that MODE gives better performance than MUSIC and NSF and achieves the CRB as $\Delta\theta$ increases. We have also found that the CRBs for COLD array are much lower than those for CCD array, especially when θ approaches 90° .

Second, we consider how the polarization separation affects the estimator performance. Consider the case where two incident signals with a correlation coefficient 0.99 arrive from angles $\theta_1 = 50^\circ$ and $\theta_2 = 70^\circ$. We assume that the corresponding ellipticity angles are $\alpha_1 = 45^\circ - \Delta\alpha$ and $\alpha_2 = 45^\circ$ and the orientation angles are $\beta_1 = \beta_2 = 0^\circ$. The polarization separation between the two polarization states is $2\Delta\alpha$. Figure 5.4 shows the RMSEs of the direction and polarization estimates as a

function of $\Delta\alpha$ for the second signal. Note that MODE performs better than MUSIC and NSF since the signals are highly correlated. Since the second signal arrives from a large angle away from the broadside of the array, the performance of the COLD array is again much better than that of the CCD array.

Third, we consider how the correlation coefficient between the two incident signals affects both the direction and polarization estimates. Figure 5.5 shows the RMSEs of the direction and polarization estimates as a function of the correlation coefficient. The two signals arrive from $\theta_1 = -6^\circ$ and $\theta_2 = 6^\circ$ and we have $\alpha_1 = \alpha_2 = 45^\circ$ and $\beta_1 = \beta_2 = 0^\circ$. We note that for highly correlated or coherent signals, the performance of MODE is much better than that of MUSIC and NSF.

Fourth, we consider the performance of the estimators as a function of SNR. In Figure 5.6, we consider the case where $\theta_1 = 50^\circ$, $\theta_2 = 70^\circ$, $\alpha_1 = \alpha_2 = 0^\circ$, $\beta_1 = \beta_2 = 0^\circ$, correlation coefficient = 0.99, and $N = 400$. In this case, the advantage of MODE over MUSIC and NSF is increasingly evident at low SNR. We note again that the COLD array gives much lower CRB than the CCD array, especially for angle estimation.

Fifth, we consider the performance of the MODE estimator as a function of the number of snapshots N . We consider the case where $\theta_1 = 50^\circ$, $\theta_2 = 70^\circ$, $\alpha_1 = \alpha_2 = 45^\circ$, $\beta_1 = \beta_2 = 0^\circ$, correlation coefficient = 0.99, and SNR = 10 dB. It can be seen from Figure 5.7 that although MODE is an asymptotically (for large N) statistically efficient estimator, MODE estimates achieve the CRB even for very moderate N . MODE again performs better than MUSIC and NSF.

Finally, we remark that although we make the Gaussian noise assumption in the problem formulation, the Gaussian noise assumption is *not* critical to the algorithms. As a matter of fact, the asymptotic distributional properties of all of the algorithms are most likely to remain the same even in the non-Gaussian noise case. However, of course, in the latter case, MODE is no longer asymptotically statistically efficient and the CRB given in (5.55) is no longer the true CRB matrix, but rather the asymptotic theoretical performance (TP) of MODE. Figure 5.8 shows an example when the Gaussian noise assumption is violated. Figure 5.8 is the same as Figure 5.7 except that the noise is the contaminated Gaussian noise [105]. More specifically, the probability density function of $\mathbf{n}(t)$ has the form $(1 - \epsilon)\mathcal{N}(\mathbf{0}, \sigma^2\mathbf{I}) + \epsilon\mathcal{N}(\mathbf{0}, 9\sigma^2\mathbf{I})$, where $\mathcal{N}(\mathbf{a}, \mathbf{B})$ denotes the Gaussian probability density function with mean $\mathbf{a}$ and covariance matrix $\mathbf{B}$. The SNR here is defined as $-10\log_{10}[\sigma^2(1 + 8\epsilon)]$ dB. We used $\epsilon = 0.7$ in the example. Note that MODE can also achieve its asymptotic theoretical performance for moderately large N for this case and MODE has the best performance.

5.6 Conclusions

We have presented a Co-centered Orthogonal Loop and Dipole (COLD) array. We have shown that with the COLD array, both angle and polarization estimation performance can be significantly improved as compared to using a similar crossed dipole array. We have also described how to use the asymptotically statistically efficient MODE algorithm for both angle and polarization estimation with the COLD

array. Numerical examples have been given to show that MODE gives more accurate angle and polarization estimates than MUSIC and NSF, especially for highly correlated or coherent signals.

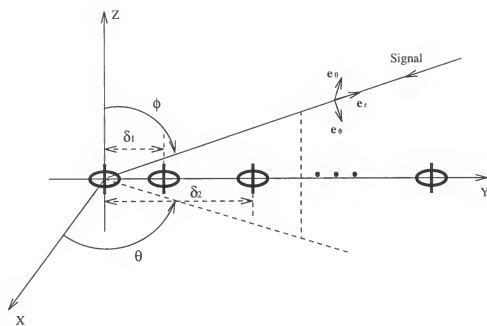
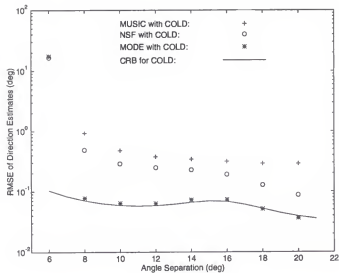
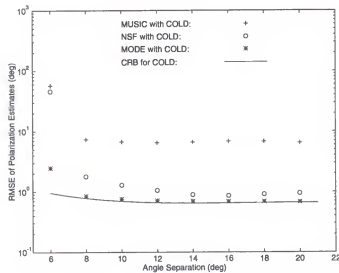


Figure 5.1. A linear COLD array.

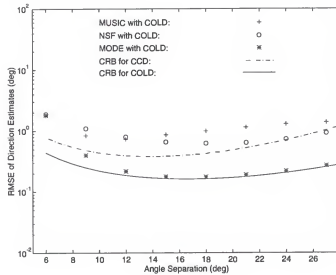


(a)

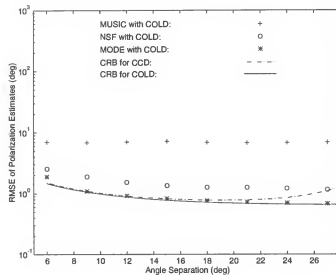


(b)

Figure 5.2. Root-mean-squared errors (RMSEs) of estimates versus $\Delta\theta$ for the first of the two signals when $\theta_1 = -\Delta\theta/2$, $\theta_2 = +\Delta\theta/2$, $\alpha_1 = \alpha_2 = 45^\circ$, $\beta_1 = \beta_2 = 0^\circ$, correlation coefficient = 0.99, $N = 400$, and SNR = 10 dB. (The CRBs for the CCD array nearly coincide with those for the COLD array.) (a) Direction estimates. (b) Polarization estimates.

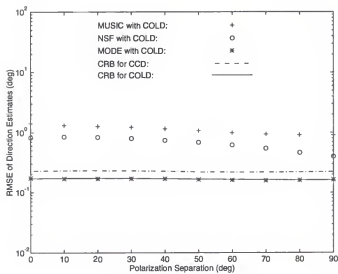


(a)

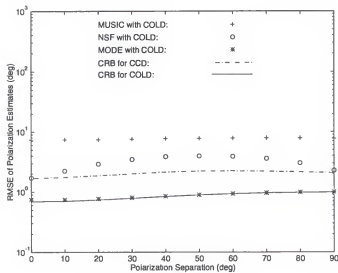


(b)

Figure 5.3. Root-mean-squared errors (RMSEs) of estimates versus $\Delta\theta$ for the second of the two signals when $\theta_1 = 50^\circ$, $\theta_2 = 50^\circ + \Delta\theta$, $\alpha_1 = \alpha_2 = 0^\circ$, $\beta_1 = \beta_2 = 0^\circ$, correlation coefficient = 0.99, $N = 400$, and SNR = 10 dB. (a) Direction estimates. (b) Polarization estimates.

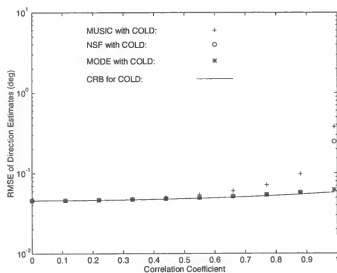


(a)

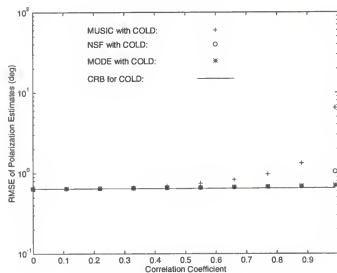


(b)

Figure 5.4. Root-mean-squared errors (RMSEs) of estimates versus $\Delta\alpha$ for the second of the two signals when $\theta_1 = 50^\circ$, $\theta_2 = 70^\circ$, $\alpha_1 = 45^\circ - \Delta\alpha$ and $\alpha_2 = 45^\circ$, $\beta_1 = \beta_2 = 0^\circ$, correlation coefficient = 0.99, $N = 400$, and SNR = 10 dB. (a) Direction estimates. (b) Polarization estimates.

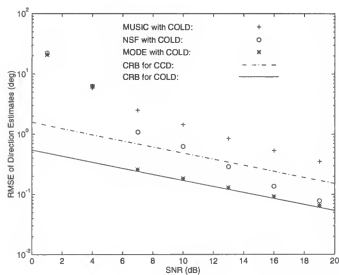


(a)

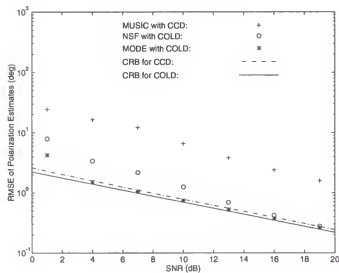


(b)

Figure 5.5. Root-mean-squared errors (RMSEs) of estimates versus source correlation coefficient for the first of the two signals when $\theta_1 = -6^\circ$, $\theta_2 = 6^\circ$, $\alpha_1 = \alpha_2 = 45^\circ$, $\beta_1 = \beta_2 = 0^\circ$, $N = 400$, and SNR = 10 dB. (The CRBs for the CCD array nearly coincide with those for the COLD array.) (a) Direction estimates. (b) Polarization estimates.

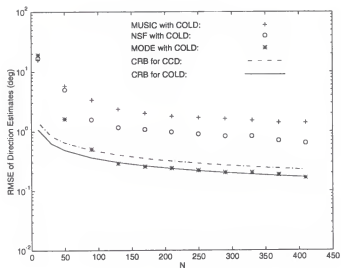


(a)

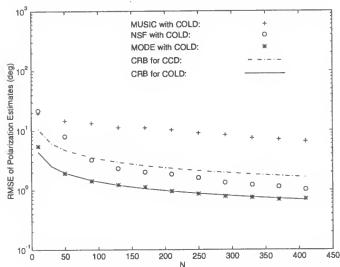


(b)

Figure 5.6. Root-mean-squared errors (RMSEs) of estimates versus SNR for the second of the two signals when $\theta_1 = 50^\circ$, $\theta_2 = 70^\circ$, $\alpha_1 = \alpha_2 = 0^\circ$, $\beta_1 = \beta_2 = 0^\circ$, correlation coefficient = 0.99, and $N = 400$. (a) Direction estimates. (b) Polarization estimates.

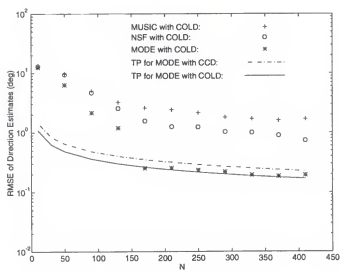


(a)

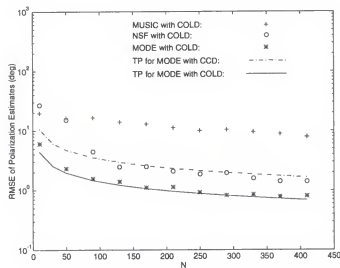


(b)

Figure 5.7. Root-mean-squared errors (RMSEs) of estimates versus N for the second of the two signals when $\theta_1 = 50^\circ$ and $\theta_2 = 70^\circ$, $\alpha_1 = \alpha_2 = 45^\circ$, $\beta_1 = \beta_2 = 0^\circ$, correlation coefficient = 0.99, and SNR = 10 dB. (a) Direction estimates. (b) Polarization estimates.



(a)



(b)

Figure 5.8. Root-mean-squared errors (RMSEs) of estimates versus N for the second of the two signals in the presence of contaminated Gaussian noise when $\theta_1 = 50^\circ$ and $\theta_2 = 70^\circ$, $\alpha_1 = \alpha_2 = 45^\circ$, $\beta_1 = \beta_2 = 0^\circ$, correlation coefficient = 0.99, and SNR = 10 dB. (a) Direction estimates. (b) Polarization estimates.

CHAPTER 6

PARAMETER ESTIMATION USING RELAX WITH A COLD ARRAY

6.1 Introduction

The RELAX algorithm described in [45] is both conceptually and computationally simple: its implementation mainly requires a sequence of fast Fourier transforms (FFTs). However, the RELAX algorithm has been used only for either temporal spectral analysis [45] or angle and waveform estimation with a uniform linear array [35]. In this Chapter, we derive a RELAX approach for angle, polarization and waveform estimation of completely polarized narrowband electromagnetic plane waves arriving at a polarization sensitive array, namely a uniform linear COLD (co-centered orthogonal loop and dipole) array as described in Chapter 5. This new RELAX estimator also requires only a sequence of FFTs. The performance of this RELAX estimator will be compared with that of the MODE estimator in Chapter 5 via numerical examples.

6.2 Problem Formulation

The COLD array we consider herein is a uniform linear COLD array with spacing δ between the adjacent sensor pairs, as shown in Figure 6.1. For a given signal impinging on the array from angles θ and ϕ , where θ and ϕ denote the azimuth and elevation angles, respectively, with its state of polarization specified by constants γ

and η , the electric field components are given by

$$E_\theta = E \cos \gamma, \quad (6.1)$$

$$E_\phi = E \sin \gamma e^{j\eta}, \quad (6.2)$$

where E denotes the amplitude of the incident signal. The γ and η can be used to compute α and β [31, 40], which are the ellipticity and orientation angles of the polarization ellipse, respectively. An incoming signal characterized by $(\theta, \phi, \gamma, \eta, E)$ produces a signal vector in the COLD pair centered at $y = (l - 1)\delta$ as follows:

$$\mathbf{z}_l(t) = \begin{bmatrix} x_l(t) \\ x_d(t) \end{bmatrix} = \boldsymbol{\mu}_{COLD} E s_o(t) e^{j \frac{2\pi(l-1)\delta}{\lambda_0} \sin \theta}, \quad (6.3)$$

where λ_0 is the wavelength of the signal, and

$$\boldsymbol{\mu}_{COLD} = \begin{bmatrix} j \frac{2\pi A_{sl}}{\lambda_0} \sin \phi \cos \gamma \\ -L_{sd} \sin \phi \sin \gamma e^{j\eta} \end{bmatrix}, \quad (6.4)$$

with L_{sd} and A_{sl} denoting the length of the dipole and the area of the loop, respectively.

For the case that the antennas and the incident signals are co-planar, i.e., $\phi = 90^\circ$, ((6.4) becomes

$$\boldsymbol{\mu}_{COLD} = \begin{bmatrix} V_\theta \cos \gamma \\ V_\phi \sin \gamma e^{j\eta} \end{bmatrix}, \quad (6.5)$$

where

$$V_\theta = j \frac{2\pi A_{sl}}{\lambda_0}, \quad (6.6)$$

$$V_\phi = -L_{sd}. \quad (6.7)$$

Note that V_θ and V_ϕ represent the complex voltages induced at the loop and dipole outputs by a signal with a unit electric field parallel to the loops and dipoles, respectively. Let $s(t) = Es_o(t)V_\theta \cos \gamma$. The $\mathbf{z}_l(t)$ in (6.3) can be rewritten as

$$\mathbf{z}_l(t) = \mathbf{u}s(t)e^{j\frac{2\pi(l-1)\delta}{\lambda_0}\sin\theta}, \quad (6.8)$$

where

$$\mathbf{u} = \begin{bmatrix} 1 \\ \frac{V_\phi}{V_\theta} \tan \gamma e^{j\eta} \end{bmatrix} \equiv \begin{bmatrix} 1 \\ r \end{bmatrix}. \quad (6.9)$$

Assume that K signals, specified by incident angles θ_k , and polarization states r_k , $k = 1, 2, \dots, K$, are incident on the COLD array. In addition, we assume a thermal noise voltage vector $\mathbf{n}_l(t)$ is present at each output vector $\mathbf{z}_l(t)$. The $\{\mathbf{n}_l(t)\}_{l=1}^L$ are assumed to be zero-mean random vectors that are statistically dependent of each other and to have an unknown covariance matrix.

Under these assumptions, the total output vector received by a uniform linear array consisting of L pairs of COLD can be written as:

$$\mathbf{z}(t) = \sum_{k=1}^K \mathbf{z}_k(t) + \mathbf{n}(t) = \sum_{k=1}^K \begin{bmatrix} \mathbf{z}_{sl,k}(t) \\ \mathbf{z}_{sd,k}(t) \end{bmatrix} + \mathbf{n}(t), \quad (6.10)$$

where $\mathbf{z}_{sl,k}(t)$, the output vector from the L small loops by the k th signal, is given by

$$\mathbf{z}_{sl,k}(t) = \mathbf{a}(\theta_k) s_k(t), \quad (6.11)$$

and $\mathbf{z}_{sd,k}(t)$, the output vector from the L small dipoles by the k th signal, is given by

$$\mathbf{z}_{sd,k}(t) = r_k \mathbf{a}(\theta_k) s_k(t), \quad (6.12)$$

with $\mathbf{a}(\theta_k)$, $k = 1, 2, \dots, K$, being an $L \times 1$ vector for the k th signal. $\mathbf{a}(\theta_k)$ has the form

$$\mathbf{a}(\theta_k) = \begin{bmatrix} 1 & e^{j\frac{2\pi d}{\lambda_0} \sin \theta_k} & \dots & e^{j\frac{2\pi d}{\lambda_0} (L-1) \sin \theta_k} \end{bmatrix}^T, \quad k = 1, 2, \dots, K. \quad (6.13)$$

$\mathbf{n}(t)$ is a zero-mean random vector with an unknown covariance matrix $\mathbf{Q}$. We define the angle and polarization vectors as:

$$\boldsymbol{\theta} = \begin{bmatrix} \theta_1 & \theta_2 & \dots & \theta_K \end{bmatrix}^T, \quad (6.14)$$

$$\mathbf{r} = \begin{bmatrix} r_1 & r_2 & \dots & r_K \end{bmatrix}^T. \quad (6.15)$$

Then the received signal vector can be written as:

$$\mathbf{z}(t) = \overline{\mathbf{A}}(\boldsymbol{\theta}, \mathbf{r})\mathbf{s}(t) + \mathbf{n}(t), \quad (6.16)$$

where $\overline{\mathbf{A}}(\boldsymbol{\theta}, \mathbf{r})$ is the $2L \times K$ matrix

$$\begin{aligned} \overline{\mathbf{A}}(\boldsymbol{\theta}, \mathbf{r}) &= \begin{bmatrix} \bar{\mathbf{a}}(\theta_1, r_1) & \bar{\mathbf{a}}(\theta_2, r_2) & \cdots & \bar{\mathbf{a}}(\theta_K, r_K) \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{a}(\theta_1) & \mathbf{a}(\theta_2) & \cdots & \mathbf{a}(\theta_K) \\ r_1\mathbf{a}(\theta_1) & r_2\mathbf{a}(\theta_2) & \cdots & r_K\mathbf{a}(\theta_K) \end{bmatrix}, \end{aligned} \quad (6.17)$$

and

$$\mathbf{s}(t) = \begin{bmatrix} s_1(t) \\ s_2(t) \\ \vdots \\ s_K(t) \end{bmatrix}, \quad \mathbf{n}(t) = \begin{bmatrix} \mathbf{n}_1(t) \\ \mathbf{n}_2(t) \\ \vdots \\ \mathbf{n}_L(t) \end{bmatrix}. \quad (6.18)$$

Assume that the element signals are sampled at N distinct times $t = 1, 2, \dots, N$. The random noise vectors $\mathbf{n}(t)$ at different sample times are assumed to be independent of each other. The problem of interest herein is to determine the azimuth arrival angles θ_k , the states of polarization r_k (described by (γ_k, η_k) or (α_k, β_k)), and the signal waveforms $s_k(t)$, $k = 1, 2, \dots, K$, $t = 1, 2, \dots, N$, from the measurements $\{\mathbf{z}(t)\}_{t=1}^N$.

6.3 Parameter Estimation with RELAX

The RELAX algorithm has been shown to be an efficient algorithm for temporal spectral analysis and angle and waveform estimation with a uniform linear array. We

describe below how the RELAX algorithm can be derived for parameter estimation with a uniform linear COLD array.

The RELAX algorithm minimizes the nonlinear least-squares (NLS) criterion for the data model in (6.16), which is given by

$$C_1[\theta, \mathbf{r}, \mathbf{s}(1), \mathbf{s}(2), \dots, \mathbf{s}(N)] = \sum_{t=1}^N [\mathbf{z}(t) - \overline{\mathbf{A}}(\theta, \mathbf{r})\mathbf{s}(t)]^H [\mathbf{z}(t) - \overline{\mathbf{A}}(\theta, \mathbf{r})\mathbf{s}(t)]. \quad (6.19)$$

If the noise $\mathbf{n}(t)$ were spatially white and Gaussian, minimizing (6.19) would have yielded the deterministic maximum likelihood estimates of the incident angles, polarization states, and signal waveforms [20].

The NLS criterion for estimating the k th signal parameters can be written as:

$$C_2[\theta_k, r_k, s_k(1), s_k(2), \dots, s_k(N)] = \sum_{t=1}^N [\mathbf{z}_k(t) - \overline{\mathbf{a}}(\theta_k, r_k)s_k(t)]^H [\mathbf{z}_k(t) - \overline{\mathbf{a}}(\theta_k, r_k)s_k(t)], \quad (6.20)$$

where

$$\mathbf{z}_k(t) = \mathbf{z}(t) - \sum_{i=1, i \neq k}^K \overline{\mathbf{a}}(\hat{\theta}_i, \hat{r}_i)\hat{s}_i(t), \quad (6.21)$$

and where $\bar{K}$ is an intermediate number of signals, and $\{\hat{\theta}_i, \hat{r}_i, \hat{s}_i(1), \hat{s}_i(2), \dots, \hat{s}_i(N)\}_{i=1, i \neq k}^{\bar{K}}$ are assumed to have been previously estimated.

Minimizing C_2 with respect to $s_k(1), \dots, s_k(N)$ gives their estimates

$$\begin{aligned} \hat{s}_k(t) &= \frac{\overline{\mathbf{a}}^H(\theta_k, r_k)\mathbf{z}_k(t)}{L(1 + |r_k|^2)} \Big|_{\theta_k = \hat{\theta}_k, r_k = \hat{r}_k} \\ &= \frac{\mathbf{a}^H(\theta_k)\mathbf{z}_{sl,k}(t) + r_k^* \mathbf{a}^H(\theta_k)\mathbf{z}_{sd,k}(t)}{L(1 + |r_k|^2)} \Big|_{\theta_k = \hat{\theta}_k, r_k = \hat{r}_k}, \quad t = 1, 2, \dots, N. \end{aligned} \quad (6.22)$$

The estimates of θ_k and r_k , $k = 1, 2, \dots, K$, are obtained by minimizing

$$C_3(\theta_k, r_k) = \sum_{t=1}^N \left\| \left[\mathbf{I} - \frac{\bar{\mathbf{a}}(\theta_k, r_k) \bar{\mathbf{a}}^H(\theta_k, r_k)}{L(1 + |r_k|^2)} \right] \mathbf{z}_k(t) \right\|^2, \quad (6.23)$$

or maximizing

$$C_4(\theta_k, r_k) = \frac{1}{1 + |r_k|^2} \sum_{t=1}^N \left| \bar{\mathbf{a}}^H(\theta_k, r_k) \mathbf{z}_k(t) \right|^2, \quad (6.24)$$

which can be written as:

$$C_5(\theta_k, r_k) = \frac{1}{1 + |r_k|^2} [\varepsilon_l(\theta_k) + |r_k|^2 \varepsilon_d(\theta_k) + r_k \varepsilon_{ld}(\theta_k) + r_k^* \varepsilon_{ld}^*(\theta_k)], \quad (6.25)$$

where

$$\varepsilon_l(\theta_k) = \sum_{t=1}^N \left| \mathbf{a}^H(\theta_k) \mathbf{z}_{sl,k}(t) \right|^2, \quad (6.26)$$

$$\varepsilon_d(\theta_k) = \sum_{t=1}^N \left| \mathbf{a}^H(\theta_k) \mathbf{z}_{sd,k}(t) \right|^2, \quad (6.27)$$

$$\varepsilon_{ld}(\theta_k) = \sum_{t=1}^N \{ [\mathbf{a}^H(\theta_k) \mathbf{z}_{sl,k}(t)] [\mathbf{a}^H(\theta_k) \mathbf{z}_{sd,k}(t)]^* \}. \quad (6.28)$$

Setting to zero the derivatives of C_5 with respect to the real and imaginary parts of r_k yields

$$r_k^2 \varepsilon_{ld}(\theta_k) + r_k [\varepsilon_l(\theta_k) - \varepsilon_d(\theta_k)] - \varepsilon_{ld}^*(\theta_k) = 0, \quad (6.29)$$

which gives the estimate of $\hat{r}_k$ of r_k as

$$\hat{r}_k = \frac{\varepsilon_d(\theta_k) - \varepsilon_l(\theta_k) + \sqrt{[\varepsilon_l(\theta_k) - \varepsilon_d(\theta_k)]^2 + 4|\varepsilon_{ld}(\theta_k)|^2}}{2\varepsilon_{ld}(\theta_k)} \Big|_{\theta_k = \hat{\theta}_k}. \quad (6.30)$$

Note that we chose the first root (plus sign) of Equation (6.29) as the $\hat{r}_k$ shown in (6.30) because only this root guarantees that the second derivatives of C_5 with respect to the real and imaginary parts of r_k are negative. In other words, this root assures the maximization of C_5 with respect to r_k .

Inserting (6.30) into (6.25) leads to a function $C_6(\theta_k)$, which depends on $\varepsilon_l(\theta_k)$, $\varepsilon_d(\theta_k)$ and $\varepsilon_{ld}(\theta_k)$ that depend only on θ_k . Thus the estimates $\hat{\theta}_k$ of θ_k can be obtained by

$$\hat{\theta}_k = \arg \max_{\theta_k} \{C_6(\theta_k)\}. \quad (6.31)$$

Note that since $C_6(\theta_k)$ is a combination of $\varepsilon_l(\theta_k)$, $\varepsilon_d(\theta_k)$ and $\varepsilon_{ld}(\theta_k)$, it can be efficiently computed by using FFT with the data sequences $\mathbf{z}_{sl,k}(t)$ and $\mathbf{z}_{sd,k}(t)$ padded with zeros. Hence $\hat{\theta}_k$ can be obtained as the location of the dominant peak of $C_6(\theta_k)$. Once $\hat{\theta}_k$ is determined, $\hat{r}_k$ can be readily calculated from (6.30). Once $\{\hat{\theta}_k, \hat{r}_k\}$ is determined, $\hat{s}_k(t)$ can be readily computed by using (6.22).

Assume that the number of incident signals K is already known or estimated. Then the RELAX algorithm has the following steps:

Step (1): Let $\bar{K} = 1$. Obtain $\hat{\theta}_1$, $\hat{r}_1$ and $\hat{s}_1(t)$ from $\mathbf{z}_1(t)$ as in (6.31), (6.30) and (6.22), respectively, $t = 1, 2, \dots, N$.

Step (2): Let $\bar{K} = 2$. Compute $\mathbf{z}_2(t)$ with (6.21) by using $\hat{\theta}_1$, $\hat{r}_1$ and $\hat{s}_1(t)$ obtained in Step (1), $t = 1, 2, \dots, N$. Obtain $\hat{\theta}_2$, $\hat{r}_2$ and $\hat{s}_2(t)$ from $\mathbf{z}_2(t)$ as in (6.31), (6.30) and (6.22), respectively, $t = 1, 2, \dots, N$.

Next, compute $\mathbf{z}_1(t)$ with (6.21) by using $\hat{\theta}_2$, $\hat{r}_2$ and $\hat{s}_2(t)$ and redetermine $\hat{\theta}_1$, $\hat{r}_1$ and $\hat{s}_1(n)$ from $\mathbf{z}_1(t)$ as in (6.31), (6.30) and (6.22), respectively, $t = 1, 2, \dots, N$. Iterate the previous two substeps until “practical convergence” is achieved (to be discussed later on).

Step (3): Let $\bar{K} = 3$. Compute $\mathbf{z}_3(t)$ with (6.21) by using $\{\hat{\theta}_i, \hat{r}_i, \hat{s}_i(1), \hat{s}_i(2), \dots, \hat{s}_i(N)\}_{i=1}^2$ obtained in Step (2). Obtain $\hat{\theta}_3$, $\hat{r}_3$ and $\hat{s}_3(t)$ from $\mathbf{z}_3(t)$ as in (6.31), (6.30) and (6.22), respectively, $t = 1, 2, \dots, N$.

Next, compute $\mathbf{z}_1(t)$ with (6.21) by using $\{\hat{\theta}_i, \hat{r}_i, \hat{s}_i(1), \hat{s}_i(2), \dots, \hat{s}_i(N)\}_{i=2}^3$ and redetermine $\hat{\theta}_1$, $\hat{r}_1$ and $\hat{s}_1(t)$ from $\mathbf{z}_1(t)$, $t = 1, 2, \dots, N$. Then compute $\mathbf{z}_2(t)$ with (6.21) by using $\{\hat{\theta}_i, \hat{r}_i, \hat{s}_i(1), \hat{s}_i(2), \dots, \hat{s}_i(N)\}_{i=1,3}$ and redetermine $\hat{\theta}_2$, $\hat{r}_2$ and $\hat{s}_2(t)$ from $\mathbf{z}_2(t)$, $t = 1, 2, \dots, N$.

Iterate the previous three substeps until “practical convergence” is achieved.

Steps (4) - (K): Continue similarly until $\bar{K}$ is equal to the known or estimated number of signal sources. (See [45] for a possible approach to estimate K when it is unknown.)

Final Step: Determine $\hat{\boldsymbol{\gamma}}$ and $\hat{\boldsymbol{\eta}}$ from $\hat{\mathbf{r}}$ with

$$\hat{\gamma}_k = \tan^{-1} \left(\left| \frac{\hat{r}_k V_\theta}{V_\phi} \right| \right), \quad (6.32)$$

and

$$\hat{\eta}_k = \arg \left(\frac{\hat{r}_k V_\theta}{V_\phi} \right), \quad k = 1, 2, \dots, K. \quad (6.33)$$

The “practical convergence” in the iterated substeps of the above RELAX algorithm may be determined by checking the relative change of the cost function in (6.19) between two consecutive iterations. In our numerical examples, to be presented in the next section, we terminate the iterations when the relative change is less than or equal to $\epsilon = 10^{-3}$.

6.4 Numerical Results

We present below several examples showing the performance of the RELAX algorithm for parameter estimation with the COLD array. We also compare the performance of the RELAX estimator with that of the MODE estimator described in [34]. The simulation results were obtained by using 50 Monte-Carlo simulations. In the examples, we assume that there are $K = 2$ incident signals and both signals are assumed to have the same amplitude E_k , such that $|V_\theta E_k| = |V_\phi E_k| = 1$, $k = 1, 2$. The noise $\mathbf{n}_l(t)$ is assumed to satisfy

$$\mathbf{n}_l(t) = -a_1 \mathbf{n}_{l-1}(t) + \mathbf{w}_l(t), \quad (6.34)$$

where $\mathbf{w}_l(t)$, $l = 1, 2, \dots, L$, are independently and identically distributed zero-mean complex Gaussian random vectors with covariance matrix $\sigma^2 \mathbf{I}$, with $\mathbf{I}$ denoting the identity matrix. The amplitudes of the signal waveforms are assumed to be 1. The SNR referred to below is defined by

$$\text{SNR} = 10 \log_{10} \left(\frac{1 - |a_1|^2}{\sigma^2} \right) \text{ [dB]}. \quad (6.35)$$

The array is assumed to have $L = 8$ COLD pairs that are uniformly spaced with the spacing between two adjacent COLD pairs equal to a half wavelength. The number of snapshots taken at the array output is $N = 2$. All array output vectors are zero padded to 2^{12} before using FFT in the RELAX algorithm.

The polarization estimation error is defined as the spherical distance between the two points M and $\hat{M}$ on the Poincaré sphere that represent the actual state of polarization (γ, η) and the estimated state of polarization $(\hat{\gamma}, \hat{\eta})$, respectively. Let ζ be the angular distance between M and $\hat{M}$. Then

$$\cos \zeta = \cos 2\gamma \cos 2\hat{\gamma} + \sin 2\gamma \sin 2\hat{\gamma} \cos(\eta - \hat{\eta}). \quad (6.36)$$

where ζ is always in the range $0 \leq \zeta \leq \pi$.

The waveform estimates for MODE are obtained by using

$$\hat{\mathbf{s}}(t) = [\mathbf{A}^H(\hat{\boldsymbol{\theta}}, \hat{\mathbf{r}}) \mathbf{A}(\hat{\boldsymbol{\theta}}, \hat{\mathbf{r}})]^{-1} \mathbf{A}^H(\hat{\boldsymbol{\theta}}, \hat{\mathbf{r}}) \mathbf{z}(t). \quad (6.37)$$

The root-mean-squared error (RMSE) of the k th waveform estimate is defined as

$$\sqrt{\sum_{t=1}^N |s_k(t) - \hat{s}_k(t)|^2}.$$

The first example illustrates the performance of the estimators as a function of SNR when the additive noise is spatially white, i.e., $a_1 = 0$ in (6.34). Consider the case where the two signals have different linear polarizations ($\alpha_1 = \alpha_2 = 0^\circ, \beta_1 = 0^\circ$, and $\beta_2 = 10^\circ$). The two signals are assumed to be correlated with correlation coefficient 0.99 and arrive at the array from $\theta_1 = 10^\circ$ and $\theta_2 = 22^\circ$. Figures 6.2(a), (b),

(c) show the root-mean-squared errors (RMSEs) of angle, polarization, and waveform estimates, respectively, of the first signal. (The results for the second signal are similar.) It can be seen from Figure 1 that RELAX and MODE yield similar performances for this case of spatially white noise.

The second example considers the case where the noise is spatially colored with $a_1 = -0.85e^{j\pi/3}$ in (6.34). The two incident signals are the same as those in the first example. Figure 6.3 shows the RMSEs of the parameter estimates of the first signal as a function of SNR. We note that RELAX outperforms MODE when the SNR is low. This result occurs because MODE is derived under the assumption of spatially white noise while the true noise is spatially colored and unknown.

Finally, we compare the computational complexities of RELAX and MODE. The amount of computations needed by RELAX depends on N , K , and the number of zero-paddings. RELAX can be faster than MODE for small N and K and at a moderate SNR when the number of zero-paddings required is small. For example, RELAX requires only about 25% of the amount of computations required by MODE when $N = 1$, $K = 2$, and the number of zero-paddings is 2^{12} . When $N = 2$, RELAX requires about 50% of the amount of computations needed by MODE. However, MODE can require much fewer computations than RELAX for large N and K and at a high SNR when the required number of zero-padding is large. Even so, RELAX can still be more attractive than MODE since the former can be easily implemented in hardware with simple FFT chips and in parallel.

6.5 Conclusions

A RELAX algorithm has been presented for angle, polarization, and waveform estimation of narrowband plane waves arriving at a uniform linear COLD array. The RELAX algorithm only requires a sequence of FFTs and is robust against the presence of unknown spatially colored noise. We have also presented numerical examples to demonstrate the performance of RELAX and compared it with that of MODE.

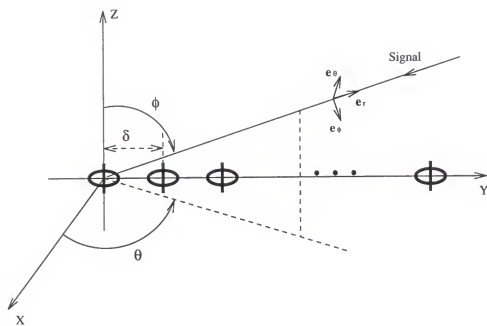
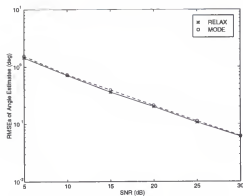
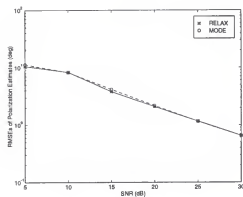


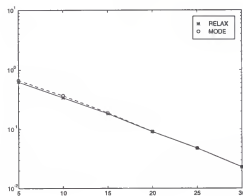
Figure 6.1. A uniform linear COLD array.



(a)

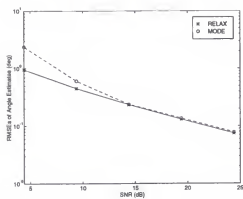


(b)

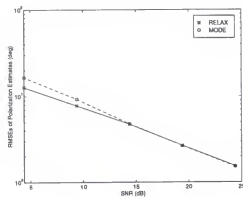


(c)

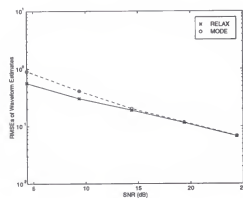
Figure 6.2. Root-mean-squared errors (RMSEs) of estimates versus SNR for the first of the two signals in the presence of white noise when $\theta_1 = 10^\circ$, $\theta_2 = 22^\circ$, $\alpha_1 = \alpha_2 = 0^\circ$, $\beta_1 = 0^\circ$, $\beta_2 = 10^\circ$, correlation coefficient = 0.99, and $N = 2$. (a) Direction estimates. (b) Polarization estimates. (c) Waveform estimates.



(a)



(b)



(c)

Figure 6.3. Root-mean-squared errors (RMSEs) of estimates versus SNR for the first of the two signals in the presence of unknown AR noise when $\theta_1 = 10^\circ$, $\theta_2 = 22^\circ$, $\alpha_1 = \alpha_2 = 0^\circ$, $\beta_1 = 0^\circ$, $\beta_2 = 10^\circ$, correlation coefficient = 0.99, and $N = 2$. (a) Direction estimates. (b) Polarization estimates. (c) Waveform estimates.

CHAPTER 7

CONCLUSIONS AND FUTURE WORK

7.1 Summary

Most existing high resolution array processing algorithms are very sensitive to the presence of unknown colored noise. The performance of those algorithms is frequently very poor when used in practical applications. To derive more robust algorithms, we may need to relax the spatially white noise assumption often used by these algorithms. A robust RELAX algorithm has been proposed in Chapter 3 for the multi-snapshot case of angle and waveform estimation of narrowband plane waves arriving at a uniform linear array. The RELAX algorithm does not assume that the noise is spatially white. The RELAX algorithm is conceptually and computationally simple; its implementation mainly requires a sequence of fast Fourier transforms. The performance of the RELAX algorithm has been evaluated and compared with those of other well-known algorithms including ESPRIT with forward/backward spatial smoothing, MODE/WSF, and AP/ANPA by the results of both simulated and experimental data. We have shown that better signal parameter estimates can be obtained by using RELAX as compared to using these other algorithms.

Since most high resolution narrowband algorithms are not readily applicable to wideband sources, focusing matrices are often needed for wideband sources and their

use can result in biases in the parameter estimates. In chapter 4 we have presented the wideband RELAX algorithm for the case of wideband sources. The wideband RELAX naturally focuses the narrowband components in the frequency domain. We have used both numerical and experimental examples to demonstrate the performance of the wideband RELAX algorithm and compared its performance with that of the CSM-ESPRIT. We have shown that the wideband RELAX algorithm can outperform the CSM-ESPRIT algorithm.

To exploit the advantages of array geometries and receiving properties of antenna sensors, we have proposed a Co-centered Orthogonal Loop and Dipole (COLD) array in Chapter 5. The Cramer-Rao bound for the COLD array has been compared with that for the crossed dipole array. We have shown that with the COLD array, both angle and polarization estimation performance can be significantly improved as compared to using a similar crossed dipole array. We have also described how to use the asymptotically statistically efficient MODE algorithm for both angle and polarization estimation with the COLD array. Numerical examples have been given to show that MODE gives more accurate angle and polarization estimates than MUSIC and NSF, especially for highly correlated or coherent signals.

A RELAX algorithm has been devised in Chapter 6 for angle, polarization, and waveform estimation of narrowband plane waves arriving at a uniform linear COLD array. This RELAX algorithm is also both conceptually and computationally simple and is robust against the violation of the spatially white noise assumption. We have presented numerical examples to demonstrate the performance of the RELAX

algorithm for angle, polarization, and waveform estimation with the COLD array and compared it with the performance of the MODE algorithm. We have shown that this RELAX can outperform MODE and yet be computationally faster than MODE.

7.2 Contributions

The main contributions made in this dissertation are:

- Developed the RELAX algorithm for angle and waveform estimation of narrow-band plane waves arriving at a uniform linear array. The RELAX algorithm is robust against the presence of unknown spatially colored noise, which occurs frequently in practical situations.
- Evaluated the performance of the narrowband RELAX algorithm and compared it with those of other well-known algorithms by using both simulated and experimental data.
- Proposed the wideband RELAX algorithm for angle estimation of wideband plane waves arriving at a uniform linear array. The wideband RELAX does not need initial estimates as other wideband algorithms require and naturally focuses the narrowband components in the spatial frequency domain. Thus the wideband RELAX can avoid the bias problem encountered by the well-known coherent subspace methods that use focusing matrices.

- Proposed a co-centered orthogonal loop and dipole (COLD) antenna array for the estimation of the parameters of completely polarized electromagnetic narrowband plane waves. Due to the array structure and superior receiving pattern of COLD, the performance of using the COLD array is greatly improved as compared to using a conventional crossed dipole array.
- Devised an efficient MODE algorithm for both angle and polarization estimation of correlated (including coherent) or uncorrelated incident signals with a COLD array.
- Devised a RELAX algorithm that can be used with a uniform linear COLD array for angle, polarization and waveform estimation of narrowband plane waves.

7.3 Future Work

The topic of polarization property of antenna sensors and electromagnetic waves and polarization estimation has been discussed by several researchers [10, 31, ?, 85, 88]. Our work presented in Chapters 5 and 6 [34, 37] is the most recent work on the topic of polarization estimation. All of these researches considered only completely polarized EM waves. A more general type of EM wave is a partially polarized EM wave, whose polarization state is a function of time. Partially polarized waves may be found in many applications such as radar and ionospheric radio [41, 42]. A large sample ML parameter estimator of partially polarized waves with a uniform linear array crossed dipole was presented in [93]. However, this estimator has the inherent

shortcoming of the crossed dipole array, i.e., the performance of the estimator is sensitive to AOAs. It is a great advantage to consider the parameter estimation of partially polarized waves with a COLD array. Certainly, it is a interesting and promising topic for future research.

The existing estimation algorithms for wideband signals have never been considered with a polarization sensitive array. The polarization estimation of wideband electromagnetic plane waves is a very challenging topic. However, it is possible to consider the wideband RELAX algorithm with a uniform COLD array for the angle and polarization of wideband plane waves. We shall leave it as yet another future topic.

APPENDIX A THE CRAMER-RAO BOUND FOR PARAMETER ESTIMATES

We provide herein a sketched derivation of the CRB for the angle and waveform estimates of the incident signals when the additive noise is a zero-mean colored Gaussian random process with an unknown covariance matrix $\mathbf{Q}$. The variables of the likelihood function of $\mathbf{y}(n)$ include the unknown elements of $\mathbf{Q}$, the incident angles, and the real and imaginary parts of the signal waveforms. Using the results in [45], we note that the CRB of the signal parameters for the colored noise case can be obtained from the CRB for the white noise case by simply replacing $\mathbf{A}$ with $\mathbf{Q}^{-1/2}\mathbf{A}$.

Let

$$\mathbf{G} = 2\mathbf{A}^H \mathbf{Q}^{-1} \mathbf{A}, \quad (\text{A.1})$$

and let

$$\Delta_n = 2\mathbf{A}^H \mathbf{Q}^{-1} \mathbf{D} \mathbf{P}_n, \quad n = 1, 2, \dots, N, \quad (\text{A.2})$$

where $\mathbf{P}_n = \text{diag}\{\mathbf{s}(n)\}$, $n = 1, 2, \dots, N$, and

$$\mathbf{D} = \left[\left. \frac{\partial \mathbf{a}(\theta)}{\partial \theta} \right|_{\theta=\theta_1}, \quad \dots, \quad \left. \frac{\partial \mathbf{a}(\theta)}{\partial \theta} \right|_{\theta=\theta_K} \right]. \quad (\text{A.3})$$

Also, let

$$\mathbf{\Gamma} = 2 \sum_{n=1}^N \text{Re} \left\{ \mathbf{P}_n^H \mathbf{D}^H \mathbf{Q}^{-1} \mathbf{D} \mathbf{P}_n \right\}, \quad (\text{A.4})$$

where $\text{Re}\{\mathbf{X}\}$ denotes the real part of $\mathbf{X}$. Define

$$\boldsymbol{\eta} = \begin{bmatrix} \text{Re}^T\{\mathbf{s}(1)\} & \text{Im}^T\{\mathbf{s}(1)\} & \cdots & \text{Re}^T\{\mathbf{s}(N)\} & \text{Im}^T\{\mathbf{s}(N)\} & \boldsymbol{\theta}^T \end{bmatrix}^T, \quad (\text{A.5})$$

where $\text{Im}\{\mathbf{X}\}$ denotes the imaginary part of $\mathbf{X}$. Then the CR-bound for $\boldsymbol{\eta}$ is

$$\text{CRB}(\boldsymbol{\eta}) = \begin{bmatrix} \text{Re}(\mathbf{G}) & -\text{Im}(\mathbf{G}) & & & & \text{Re}(\boldsymbol{\Delta}_1) \\ \text{Im}(\mathbf{G}) & \text{Re}(\mathbf{G}) & & \mathbf{0} & & \text{Im}(\boldsymbol{\Delta}_1) \\ & & \ddots & & & \vdots \\ & & & \mathbf{0} & \text{Re}(\mathbf{G}) & -\text{Im}(\mathbf{G}) & \text{Re}(\boldsymbol{\Delta}_N) \\ & & & & \text{Im}(\mathbf{G}) & \text{Re}(\mathbf{G}) & \text{Im}(\boldsymbol{\Delta}_N) \\ \text{Re}^T(\boldsymbol{\Delta}_1) & \text{Im}^T(\boldsymbol{\Delta}_1) & \cdots & \text{Re}^T(\boldsymbol{\Delta}_N) & \text{Im}^T(\boldsymbol{\Delta}_N) & & \mathbf{\Gamma} \end{bmatrix}^{-1}. \quad (\text{A.6})$$

We remark that when $\mathbf{Q} = \sigma^2 \mathbf{I}$, the results above are identical to those in [28].

APPENDIX B
PROOF OF EQUATION (5.27)

Let $\mathbf{B}$ be an $L \times (L - K)$ matrix whose columns span the null space of $\mathbf{A}^H$.

Also let a $(2L - K) \times 2L$ matrix Δ^H be defined as

$$\Delta^H = \begin{bmatrix} \mathbf{B}^H \otimes \mathbf{I} \\ \mathbf{V}^H (\mathbf{A}^\dagger \otimes \mathbf{I}) \end{bmatrix}, \quad (\text{B.1})$$

We first show that the columns of the matrix Δ span the null space of $\overline{\mathbf{A}}^H$.

Since $\mathbf{B}^H \mathbf{A} = \mathbf{0}$ and $\mathbf{v}_k^H \mathbf{u}_k = 0$, we obtain

$$\Delta^H \overline{\mathbf{A}} = \begin{bmatrix} (\mathbf{B}^H \mathbf{A}) \otimes \mathbf{I} \mathbf{U} \\ \mathbf{V}^H \mathbf{U} \end{bmatrix} = \mathbf{0}. \quad (\text{B.2})$$

This result shows that the columns of Δ belong to $\mathcal{N}(\overline{\mathbf{A}}^H)$. Moreover,

$$\begin{aligned} \Delta^H \Delta &= \begin{bmatrix} \mathbf{B}^H \otimes \mathbf{I} \\ \mathbf{V}^H (\mathbf{A}^\dagger \otimes \mathbf{I}) \end{bmatrix} \begin{bmatrix} \mathbf{B} \otimes \mathbf{I} & (\mathbf{A}^{\dagger H} \otimes \mathbf{I}) \mathbf{V} \end{bmatrix} \\ &= \begin{bmatrix} (\mathbf{B}^H \mathbf{B}) \otimes \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{V}^H [(\mathbf{A}^H \mathbf{A})^{-1} \otimes \mathbf{I}] \mathbf{V} \end{bmatrix}. \end{aligned} \quad (\text{B.3})$$

Thus Δ has full column rank $2(L - K) + K = 2L - K$. Since

$$\dim [\mathcal{N}(\overline{\mathbf{A}}^H)] = 2L - K, \quad (\text{B.4})$$

it follows that the columns of Δ span the entire null space $\mathcal{N}(\overline{\mathbf{A}}^H)$. Hence

$$\begin{aligned} \mathbf{P}_{\overline{\mathbf{A}}}^\perp &= \mathbf{P}_\Delta = \Delta(\Delta^H \Delta)^{-1} \Delta^H \\ &= (\mathbf{B} \otimes \mathbf{I})[(\mathbf{B}^H \mathbf{B})^{-1} \otimes \mathbf{I}](\mathbf{B}^H \otimes \mathbf{I}) \\ &\quad + (\mathbf{A}^{\dagger H} \otimes \mathbf{I})\mathbf{V}\left\{\mathbf{V}^H[(\mathbf{A}^H \mathbf{A})^{-1} \otimes \mathbf{I}]\mathbf{V}\right\}^{-1}\mathbf{V}^H(\mathbf{A}^\dagger \otimes \mathbf{I}). \end{aligned} \quad (\text{B.5})$$

Since $\mathbf{P}_{\mathbf{B} \otimes \mathbf{I}} = \mathbf{P}_{\overline{\mathbf{A}} \otimes \mathbf{I}}^\perp$, we conclude that Equation (5.27) must hold true.

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BIOGRAPHICAL SKETCH

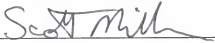
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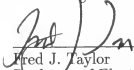
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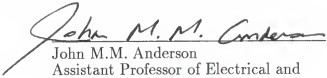
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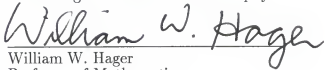
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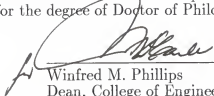

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This dissertation was submitted to the Graduate Faculty of the College of Engineering and to the Graduate School and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

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